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Reliability & Risk Engineering

Decomposition methods for large-scale linear optimization

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Energy System Optimization Workshop, TU Graz

Planning net-zero systems under climate uncertainty

Linear problems too large to fit into memory

Context and motivation

- Integrated net-zero systems at macro-scale and high spatio-temporal detail
- Seasonal storage creates interdependencies over long time-frames
- Ensure reliability under uncertain conditions (e.g., climate, technology)

Modelling approach

- Two-stage problems with system operation on the second stage
- Data-driven approach representing uncertainty as scenarios
- Either each scenario covers one year or storage levels are a first-stage decision

- Simple linear, but massive problem
- Small first-stage problem, but large second stage problems

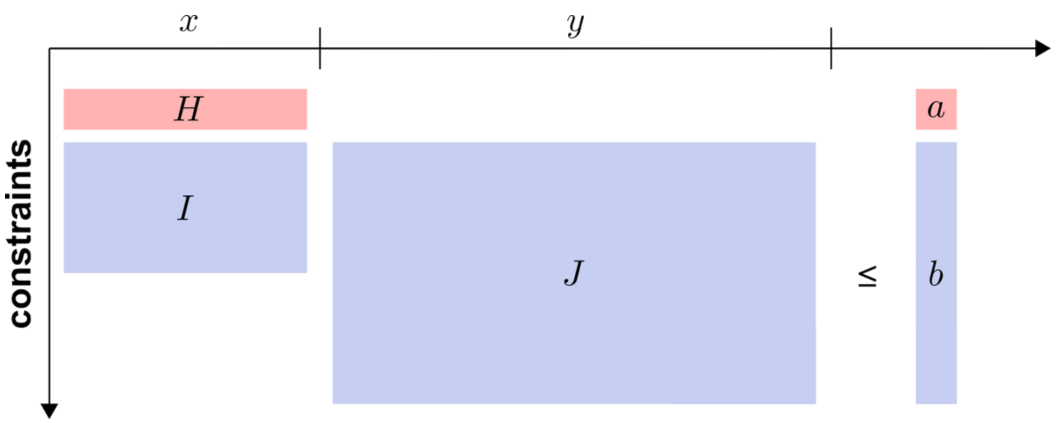
How can we further improve **regularized Benders decomposition** to solve these problems more efficiently?

Decompose optimization problems into smaller parts

Expansion problem with subordinate dispatch problems

	Matrix representation	Mathematical formulation

Monolithic



first-stage capacity decisions — second-stage operational decisions

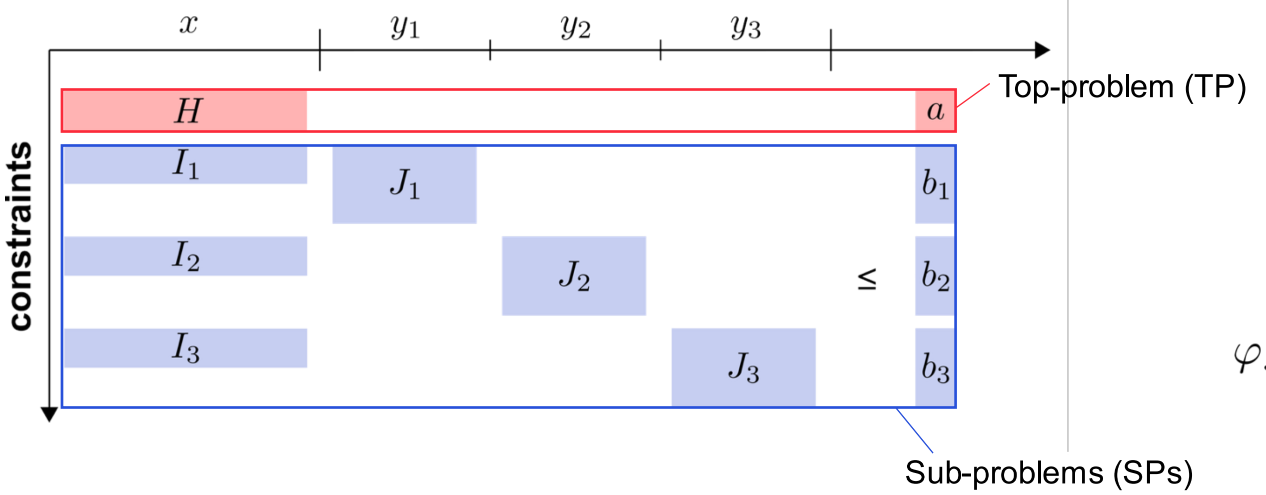
$$\min_{x,y} \; c^\top x + d^\top y$$

$$\text{s.t.} \; Hx \leq a$$

$$Ix + Jy \leq b$$

$$x \in \mathbb{R}^k, y \in \mathbb{R}^k$$

Decomposed



$$\min_{x} \; c^\top x + \sum_{s \in S} \varphi_s(x)$$

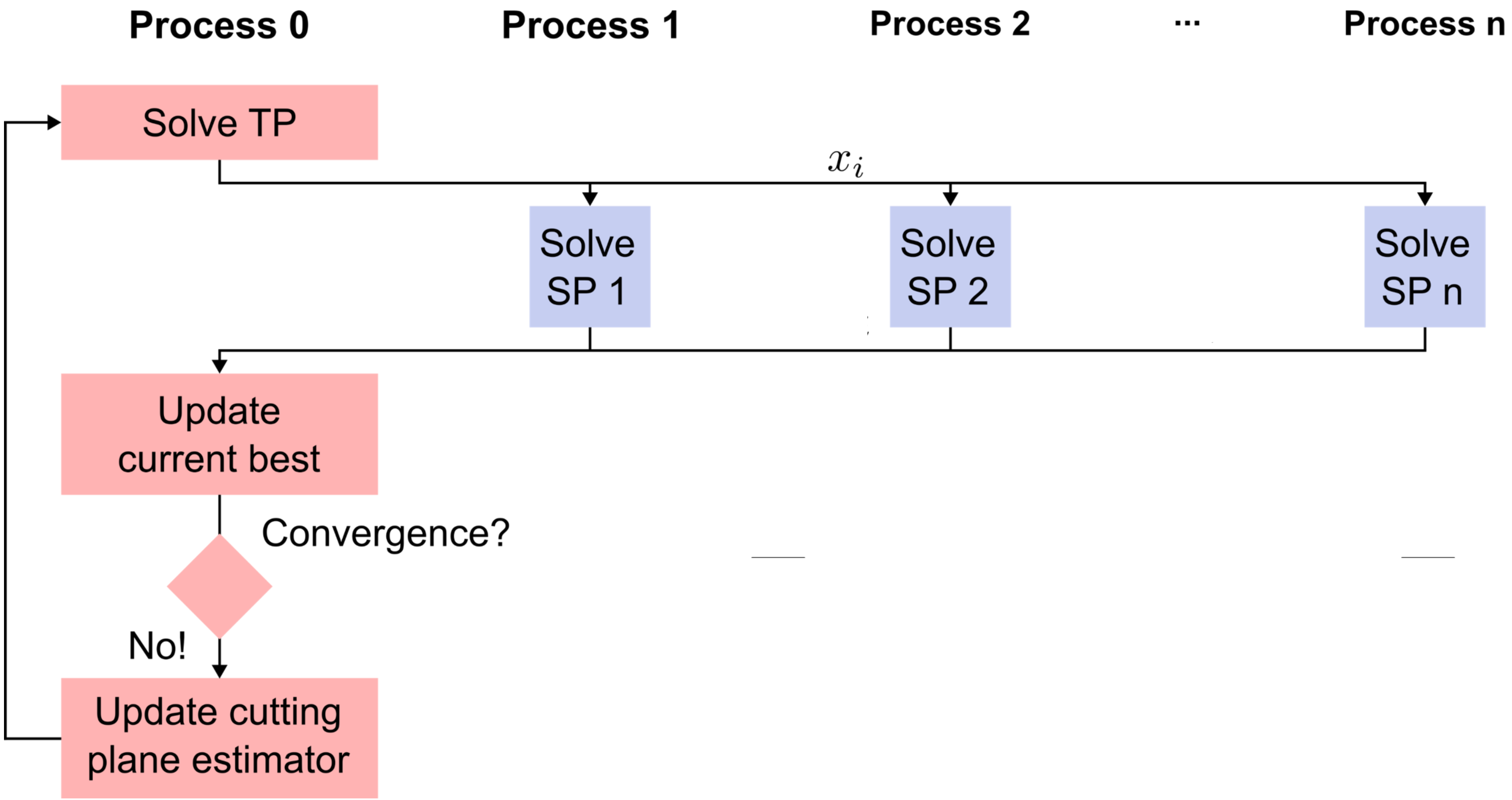
$$\text{s.t.} \; Hx \leq a$$

$$x \in \mathbb{R}^k$$

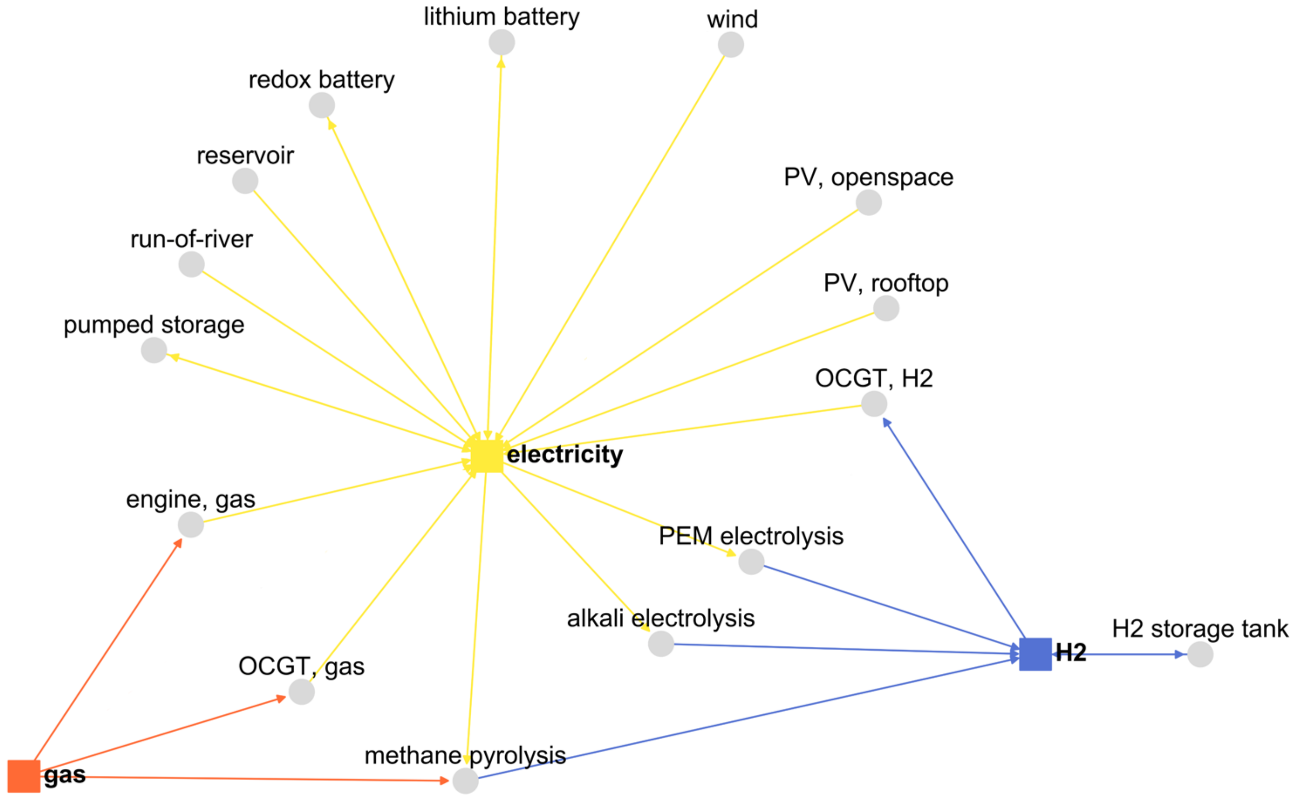
$$\varphi_s(x) := \min_{y_s \in \mathbb{R}^k} \{ d^\top y_s \mid J_s y_s \leq b_s - I_s x \}$$

Distributed Benders algorithm

Subproblems can be solved in parallel to reduce wall clock time

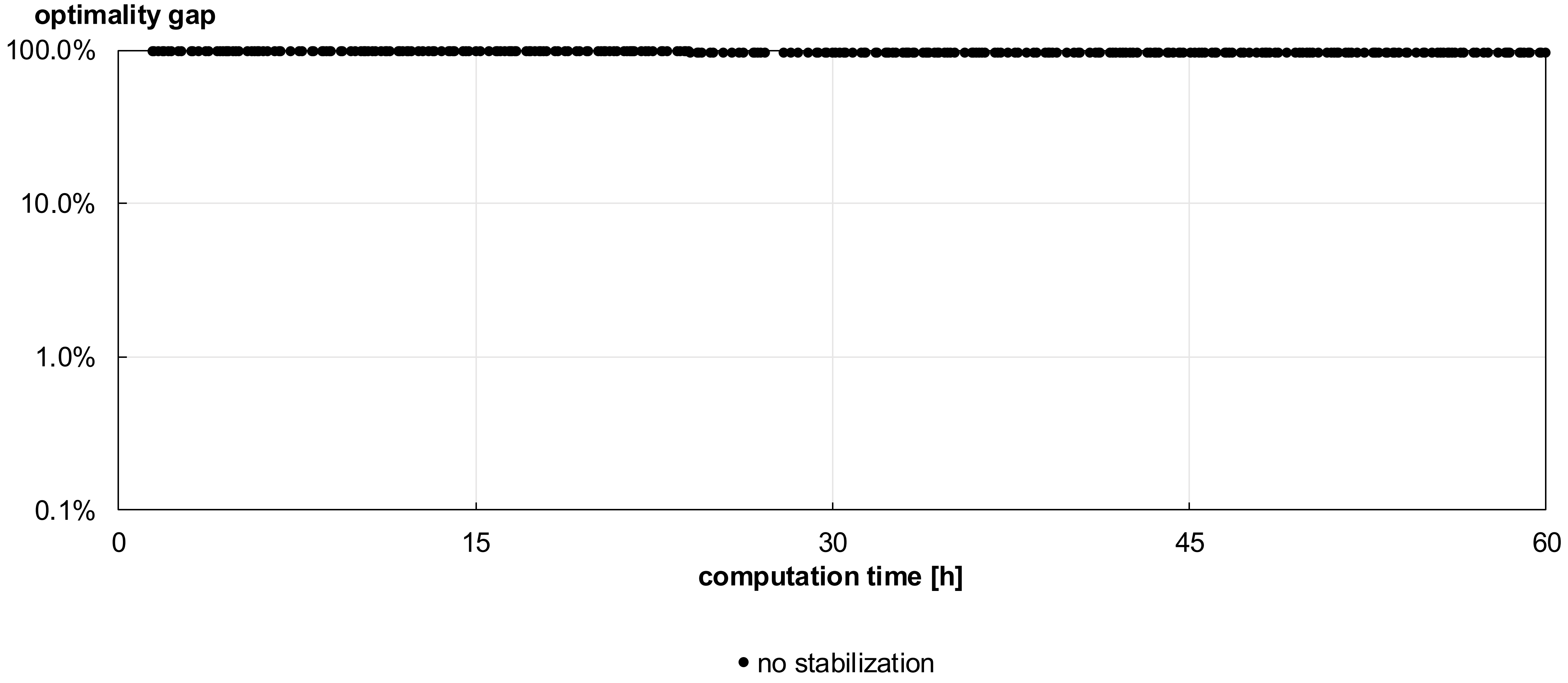


Simple benchmark model for planning under climate uncertainty
Model is already too large to fit into single-node memory



- Power sector capacity planning
- 20 scenarios for different climate years
  → perfect foresight for whole year
- 609 complicating variables
  - 264 for conversion capacity
  - 144 for storage capacity
  - 201 for transmission capacity
- Size of monolithic problem
  - $132 \cdot 10^6$ variables
  - $121 \cdot 10^6$ constraints
  - $442 \cdot 10^6$ non-zeroes

Performance of simple benchmark problem
Very slow convergence without stabilization



Combining trust-region and level bundle into new stabilization

Reduce computation time for top problem and improve convergence

Quadratic trust-region [1]

$$\min_x \; c^\top x + \sum_{s \in S} \tilde{\varphi}_s(x)$$
$$s.t. \; ||x - \hat{x}_c||_2 \le r$$

Level bundle [2]

$$\min_x \; ||x - \hat{x}_c||_2$$
$$s.t. \; c^\top x + \sum_{s \in S} \tilde{\varphi}_s(x) \le \lambda$$

Equivalent to level bundle with sub-optimal termination
→ Potentially reduce time for top-problem, since avoiding quadratic constraint

Inner-point level bundle [3]

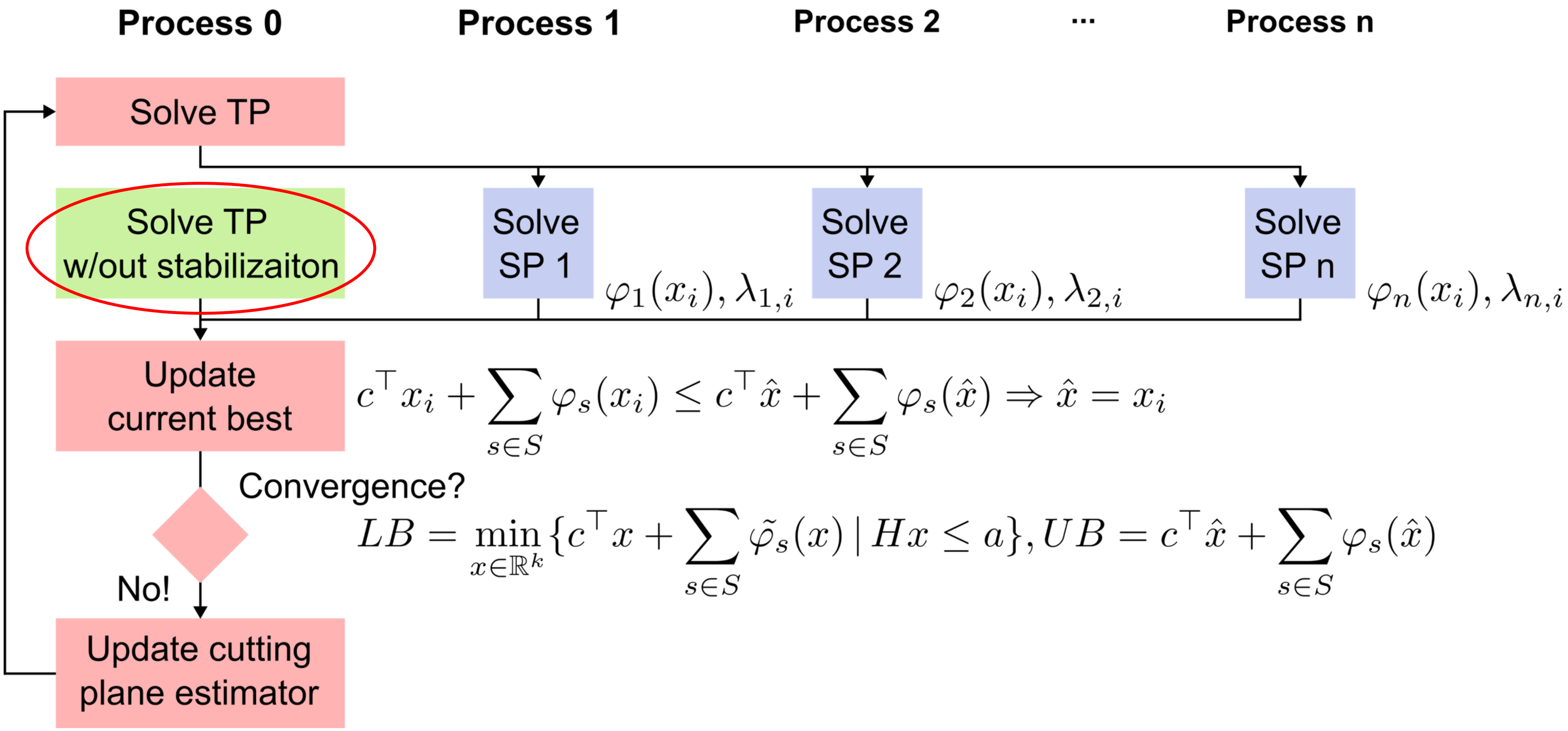
$$\min_x \; 0$$
$$s.t. \; c^\top x + \sum_{s \in S} \tilde{\varphi}_s(x) \le \lambda$$

Combined inner-point

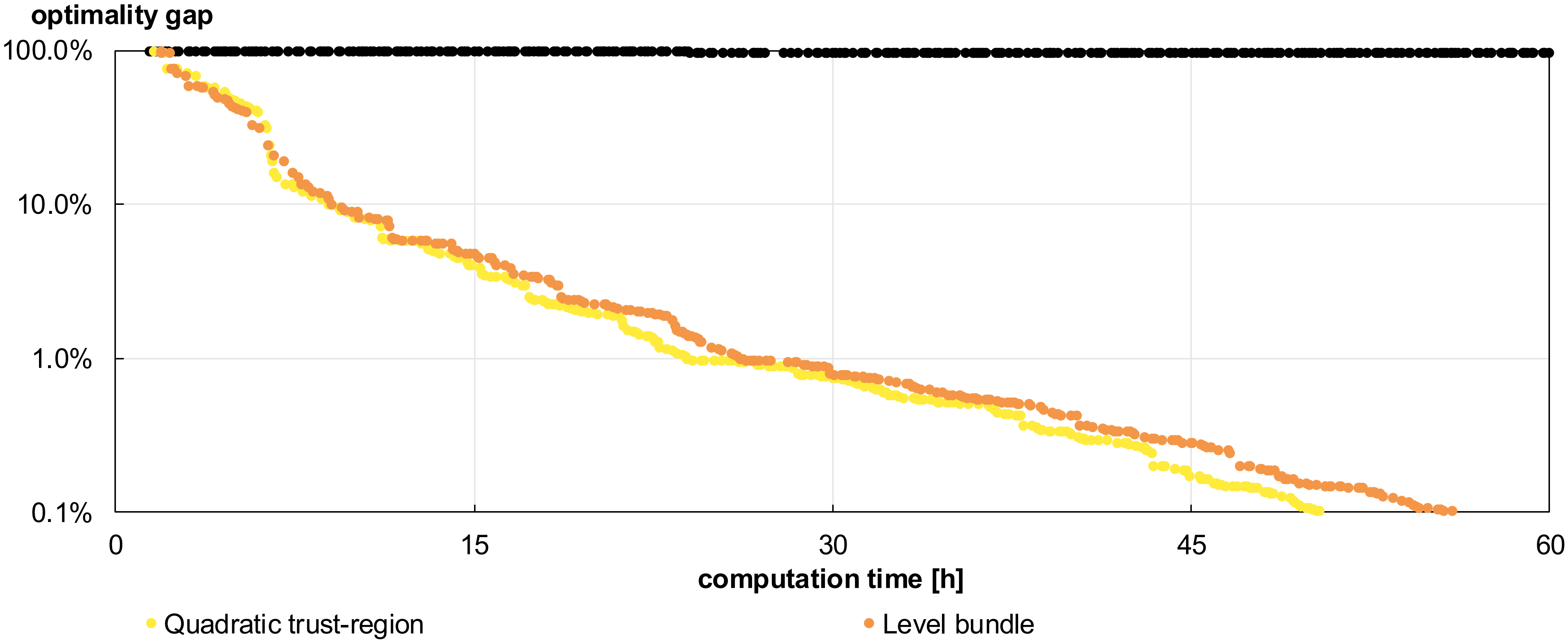
$$\min_x \; 0$$
$$s.t. \; c^\top x + \sum_{s \in S} \tilde{\varphi}_s(x) \le \lambda$$
$$||x - \hat{x}_c||_2 \le r$$

[1] Göke et al. (2024). Stabilized Benders decomposition for energy planning under climate uncertainty. *European Journal of Operational Research*, 316(1), 183-199.
[2] Ruszczynski (1986). Decomposition Methods. *Handbooks in Operations Research and Management Science*, 10, 141-211.
[3] Pecci et al. (2025), Regularized benders decomposition for high performance capacity expansion models, *IEEE Transactions on Power Systems*.

Stabilized distributed Benders algorithm
Solve top-problem w/out stabilization for lower bound

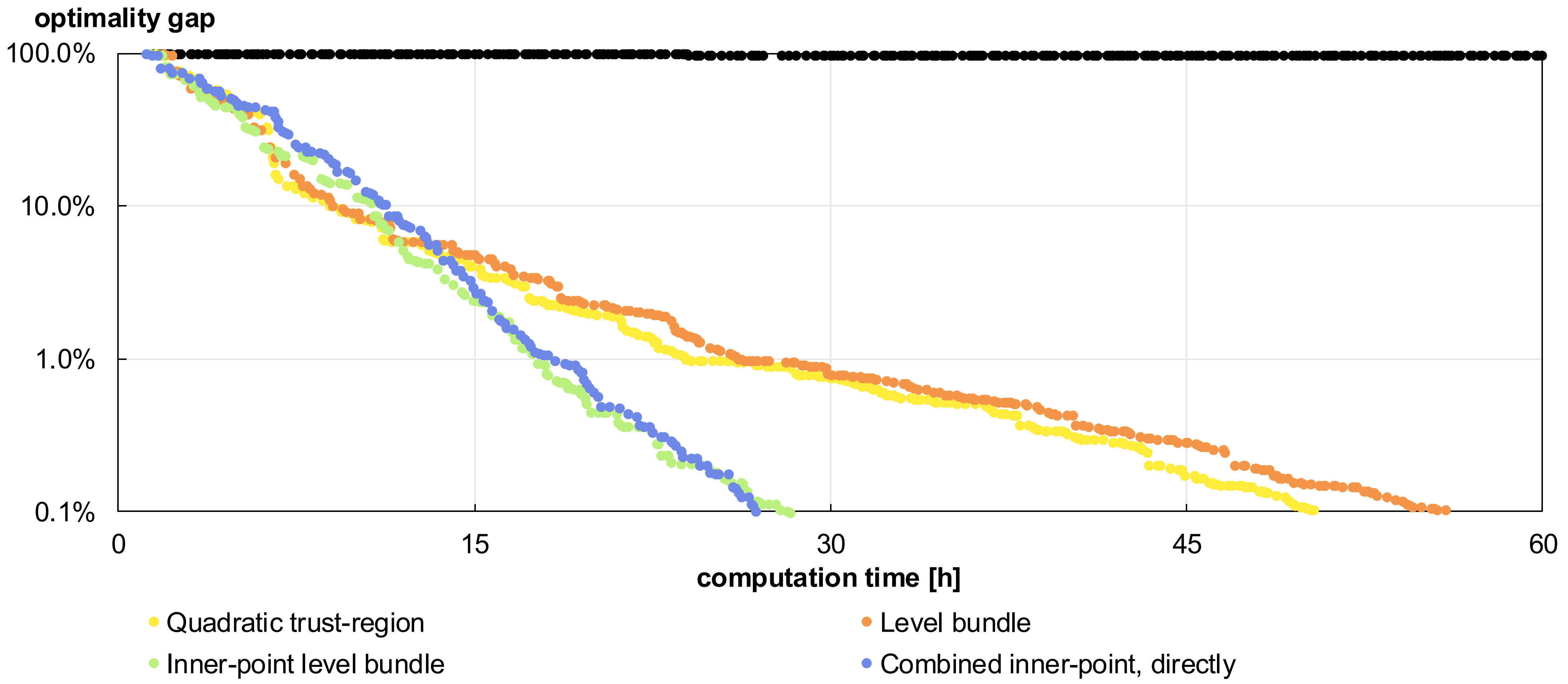


Performance of simple benchmark model
Simple stabilization achieves substantial speed-up

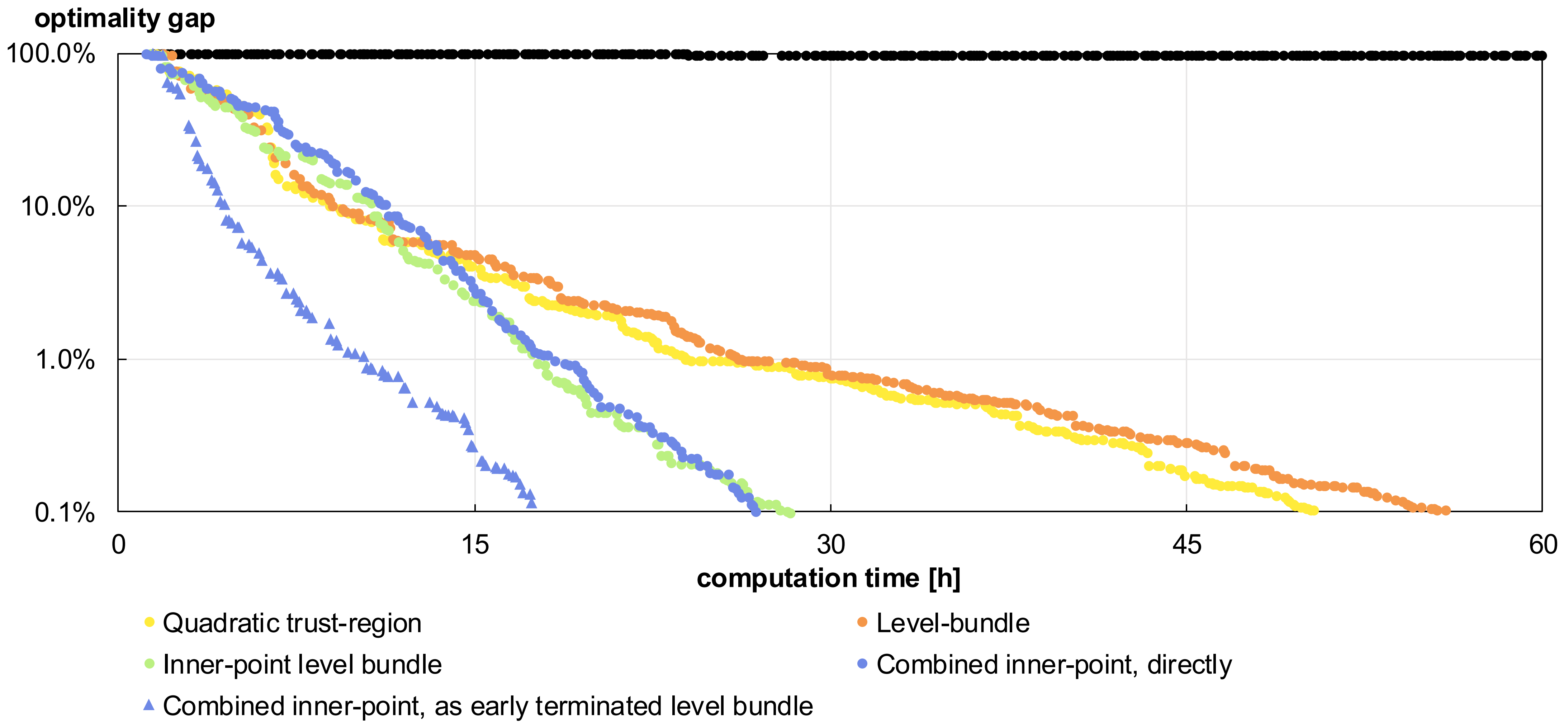


Performance of simple benchmark model

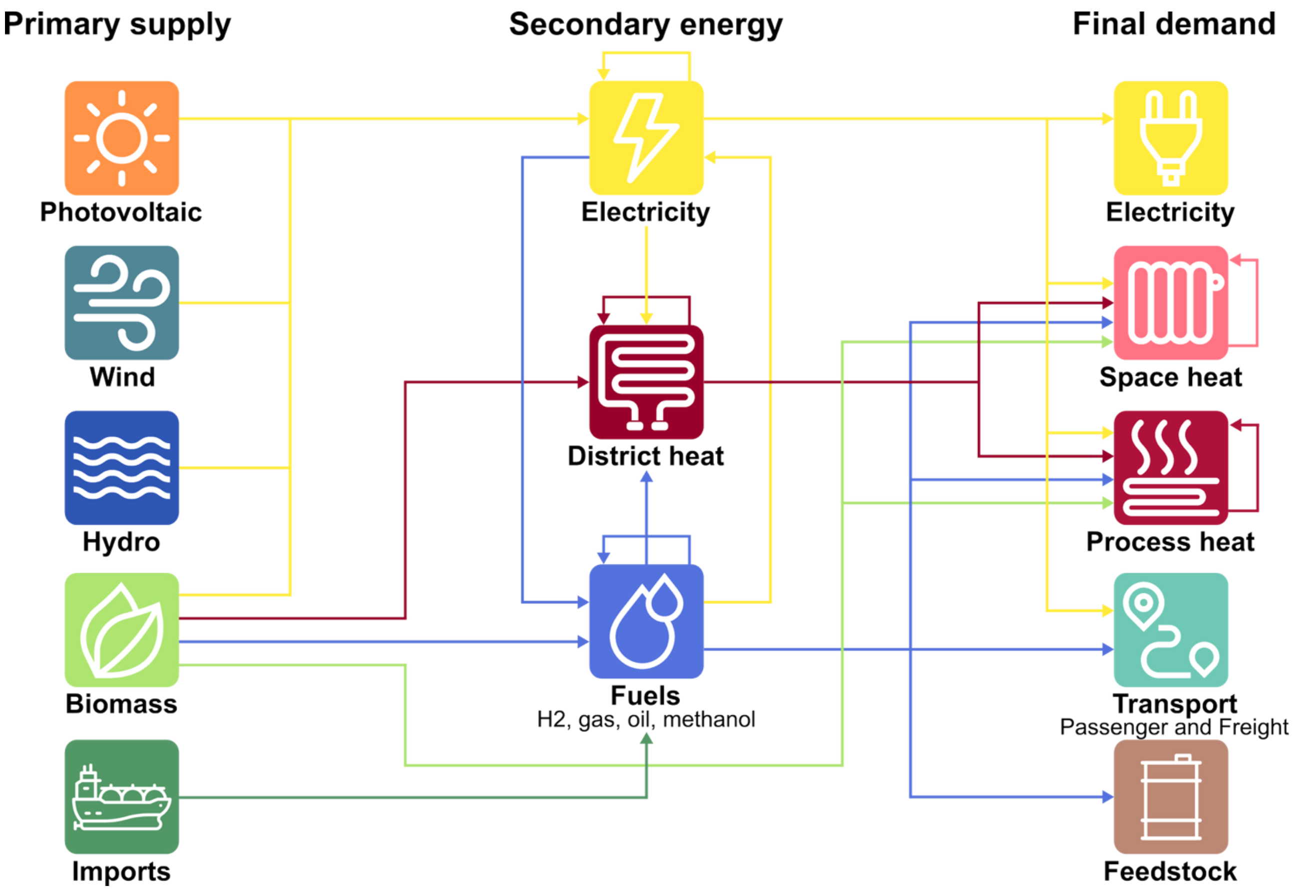
Inner-point stabilization further improves performance



Performance of simple benchmark model
Best performance for inner-point simply implemented as level bundle



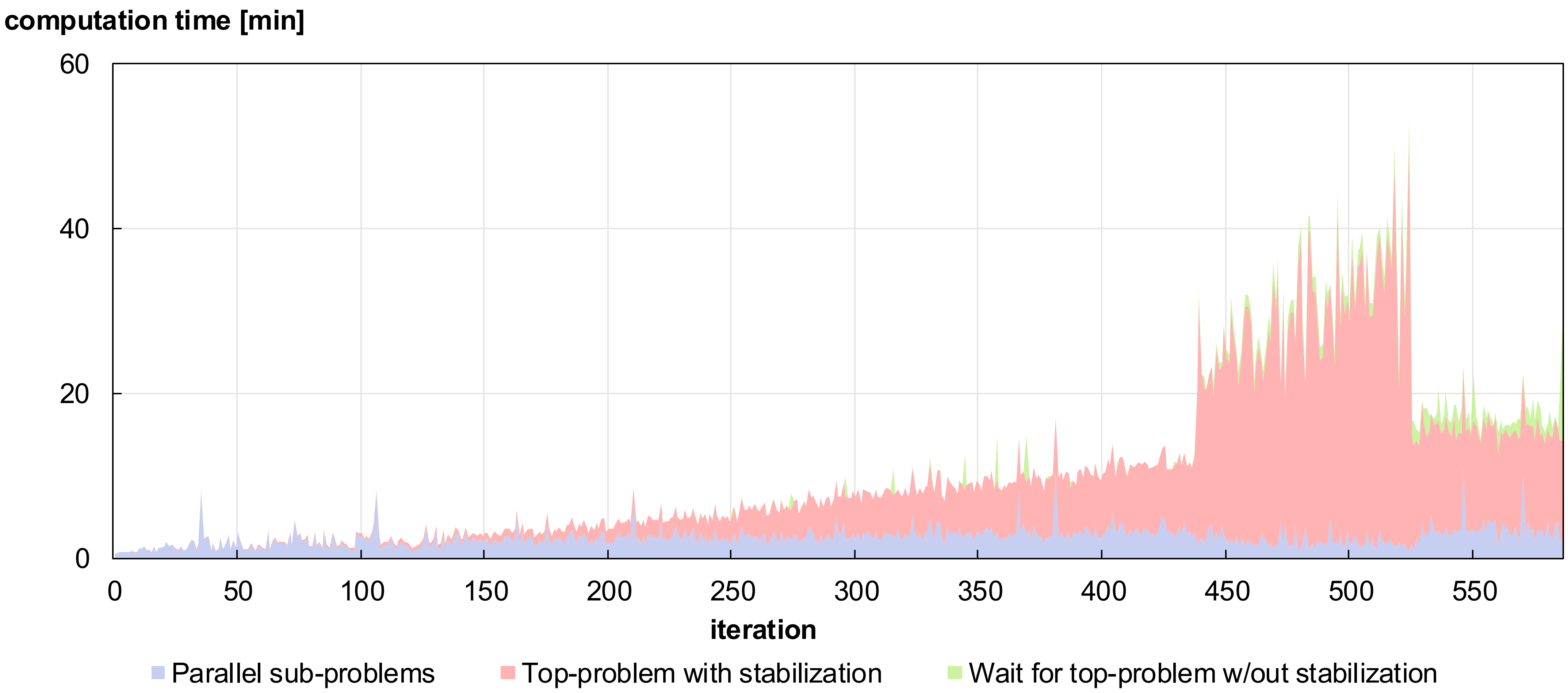
Complex applied model for planning under climate uncertainty
Scalable and robust formulation for stochastic planning



- Multi-sector capacity planning
- 32 scenarios for different climate months
  → limited foresight to one month
  → combinations cover 51’840 climate years
- 6’045 complicating variables
  - 2’081 for capacity
  - 32 for emission limits
  - 4’248 for storage levels
- Size of monolithic problem
  - $62 \cdot 10^6$ variables
  - $60 \cdot 10^6$ constraints
  - $217 \cdot 10^6$ non-zeroes

[4] Göke et al. (2025). The Liquid Buffer: Multi-Year Storage for Defossilization and Energy Security under Climate Uncertainty. *arXiv*, 10.48550/arXiv.2511.13513.

Performance of complex applied model
Addition of dense constraints as cuts escalates time for top-problem



What is next?

Conclusions

- Regularization is key to solve massive continuous problems with Benders decomposition
- Simply using the level method with sub-optimal termination outperforms much more sophisticated methods
- Increasing the number of complicating variables is critical

Outlook

- Refine strategies to decide on the structure of sub-problems
- Advance the parallelization scheme
- Replace method for suggesting new first-stage solutions

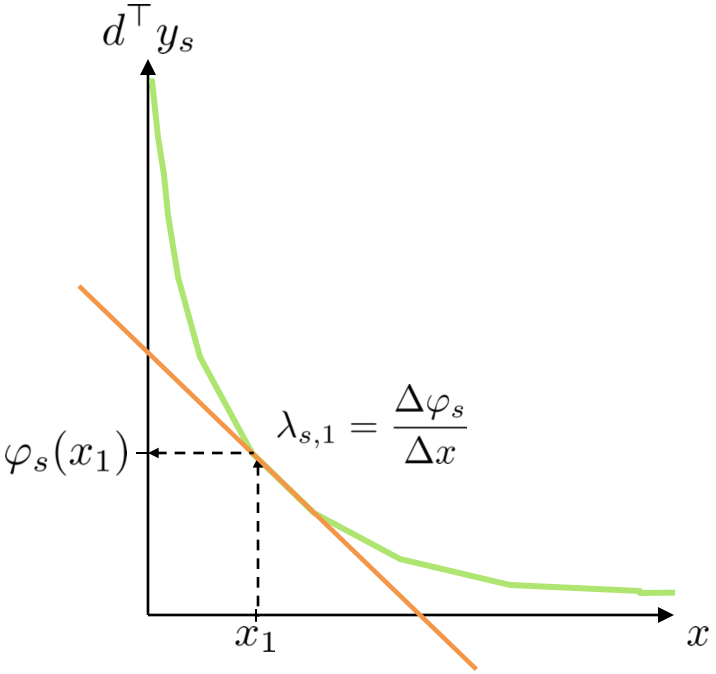
Thank you for your attention!

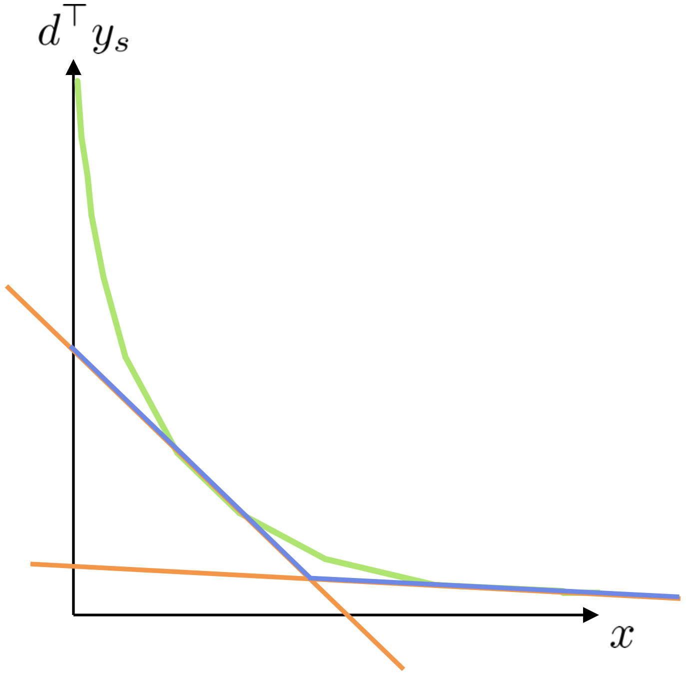
Cutting plane estimator for lower approximation of sub-problems
Solving the operation for different capacities improves the estimator

$$\varphi_s(x) := \min_{y_s \in \mathbb{R}^k} \{d^\top y_s \mid J_s y_s \leq b_s - I_s x\}$$

$$\tilde{\varphi}_s(x) = \max_{i \in N}\{d^\top y_s + \lambda_{s,i}^\top (x - \hat{x}_i)\} \Leftrightarrow \alpha_s \geq z_s + \lambda_{s,i}^\top (x - \hat{x}_i) \quad \forall i \in N$$

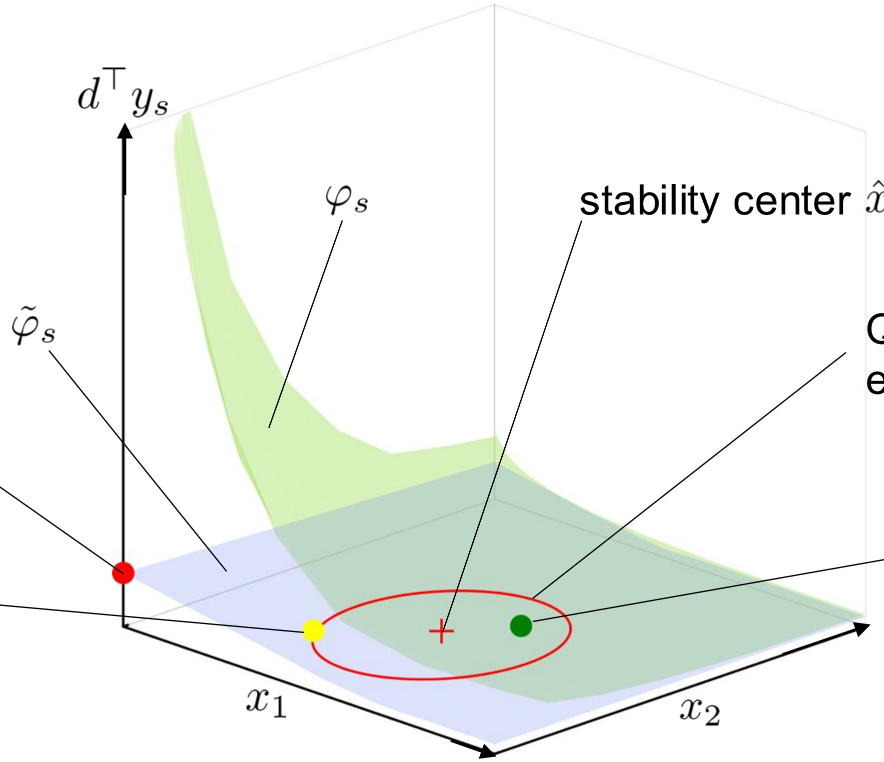
$$\tilde{\varphi}_s(x) \leq \varphi_s(x)$$





[2] Conejo et al. (2006). Decomposition Techniques in Mathematical Programming. *Springer*.

Quadratic stabilization to prevent oscillation in the top-problem



Quadratic trust-region [2]

$$\min_x \; c^\top x + \sum_{s \in S} \tilde{\varphi}_s(x)$$
$$s.t. \; ||x - \hat{x}_c||_2 \le r$$

Level bundle [3]

$$\min_x \; ||x - \hat{x}_c||_2$$
$$s.t. \; c^\top x + \sum_{s \in S} \tilde{\varphi}_s(x) \le \lambda$$

Proximal bundle [4]

$$\min_x \; c^\top x + \sum_{s \in S} \tilde{\varphi}_s(x) + \tau ||x - \hat{x}_c||_2$$

[2] Göke et al. (2024). Stabilized Benders decomposition for energy planning under climate uncertainty. *European Journal of Operational Research*, 316(1), 183-199.
[3] Ruszczynski (1986). Decomposition Methods. *Handbooks in Operations Research and Management Science*, 10, 141-211.
[4] Lemarechal et al. (1981). On a bundle algorithm for nonsmooth optimization. *Nonlinear Programming*, 4, 245-282.