Simplifying Preference Elicitation in Local Energy Markets: Combinatorial Clock Exchange

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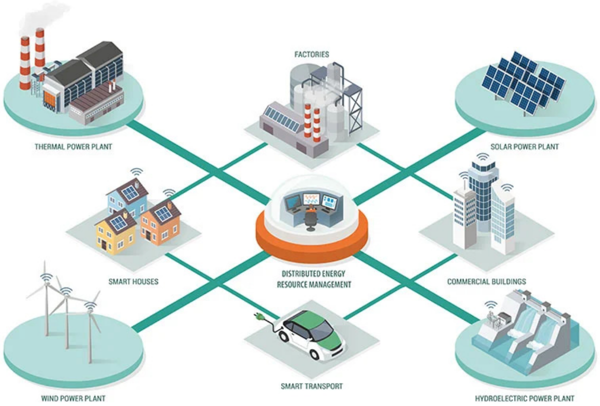
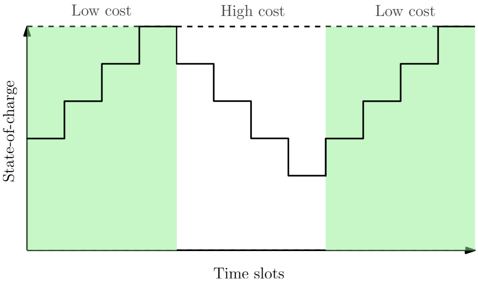
Why local energy markets?



- Responsive demand and renewable integration
- Reduced energy and infrastructure costs

Prosumers have complex preferences



- Battery energy storage system (BESS)
  - Time-coupled charging preferences

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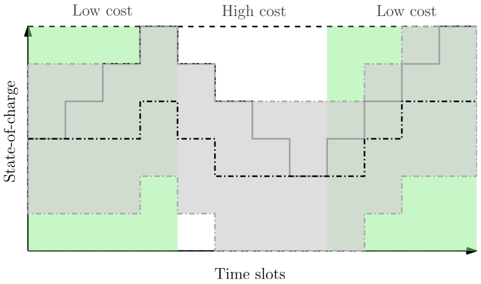

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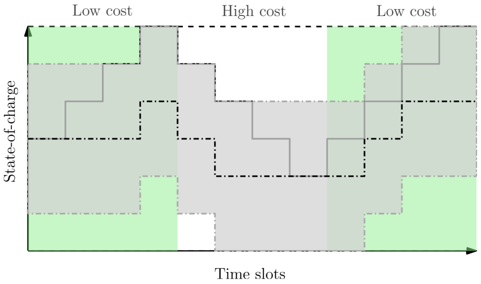

- Battery energy storage system (BESS)
  - Time-coupled charging preferences
  - Inter-product coupling: energy and flexibility
  - Additional product differentiation: e.g. green energy

Prosumers optimize their utility

- m products
- Preference over packages of products
  - Value function $v : \mathcal{X} \to \mathbb{R},\ \mathcal{X} \subset \mathbb{R}^m$
- Product prices $\boldsymbol{\lambda} \in \mathbb{R}^m$
- Package of products $\mathbf{x} \in \mathbb{R}^m$

Best-response package

$$\mathbf{x}^\star(\boldsymbol{\lambda}) \in \max_{\mathbf{x} \in \mathcal{X}} v(\mathbf{x}) - \langle \boldsymbol{\lambda}, \mathbf{x} \rangle$$

We seek prices that coordinate prosumers

Best-response package

$$\mathbf{x}^{\star}(\boldsymbol{\lambda}) \in \max_{\mathbf{x}\in\mathcal{X}} v(\mathbf{x}) - \langle \boldsymbol{\lambda}, \mathbf{x} \rangle$$

- n prosumers
- Price $\boldsymbol{\lambda}$

Production=Consumption

$$\sum_{i=1}^{n} \mathbf{x}_i^{\star}(\boldsymbol{\lambda}) = \mathbf{0}$$

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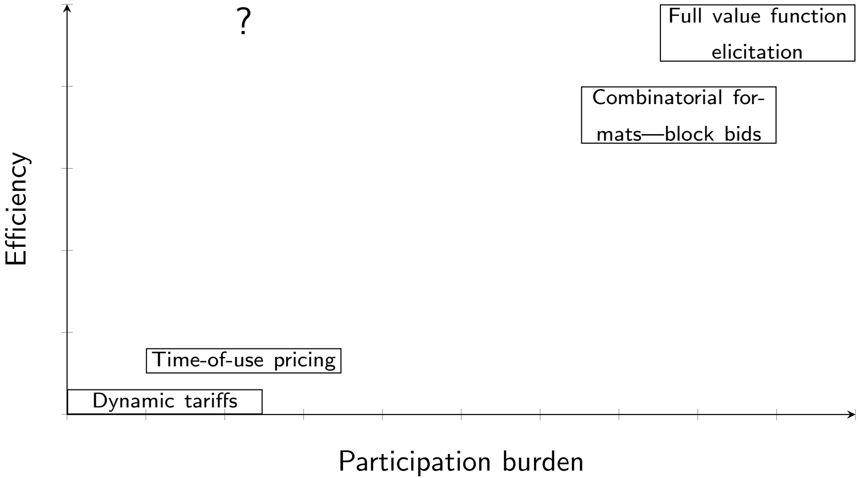
How to find such prices?

Ideal: low participation burden + high efficiency



Equilibrium prices are the solution of the dual program

- v_i: Prosumer i's value function
- $\boldsymbol{\lambda}$: Product prices

Optimal welfare

$$\max_{\mathbf{x}_i \in \mathcal{X}_i} \sum_{i=1}^{n} v_i(\mathbf{x}_i) \quad \text{s.t.} \quad \sum_{i=1}^{n} \mathbf{x}_i = \mathbf{0}$$

primal ⇄ dual

$$\boldsymbol{\lambda}^\star \in \min_{\boldsymbol{\lambda}} \underbrace{\left[\sum_{i \in \mathcal{N}} \overbrace{\max_{\mathbf{x} \in \mathcal{X}_i} v_i(\mathbf{x}) - \langle \boldsymbol{\lambda}, \mathbf{x} \rangle}^{\text{prosumer's problem}} \right]}_{g(\boldsymbol{\lambda})}$$

The dual problem is convex irrespective of primal

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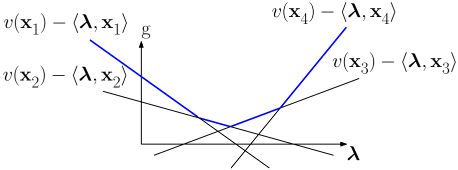
Single product case



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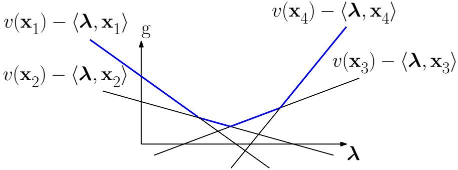

$$\nabla g(\boldsymbol{\lambda}) = -\sum_{i=1}^{n} \mathbf{x}_i^{\star}(\boldsymbol{\lambda}) \leftarrow \text{Prosumer's best response}$$

Iterative market mechanism procedure

- Guess prices $\boldsymbol{\lambda}^t$ and query prosumers' best response $\mathbf{x}_i^\star(\boldsymbol{\lambda}^t)$

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- Gradient descent step: $\boldsymbol{\lambda}^{t+1} \leftarrow \boldsymbol{\lambda}^t - \eta_t \nabla g$

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- Gradient descent step: $\boldsymbol{\lambda}^{t+1} \leftarrow \boldsymbol{\lambda}^t - \eta_t \nabla g$
- Intuitive update rule:
  - oversupply $\implies$ price $\downarrow$ overconsumption $\implies$ price $\uparrow$

Step size sequence is crucial for fast convergence

Robbins-Monro conditions on step size η_t

$$\sum_t \eta_t = \infty, \ \sum_t \eta_t^2 < \infty$$

For e.g., $\eta_t = \frac{1}{t}$

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Which step size sequence $\{\eta_t\}_{t=1}^{\infty}$ achieves quick convergence?

Elicited dataset: $\{\mathbf{x}^{\star k}, \boldsymbol{\lambda}^k\}_{k=1}^{t}$

Prosumer value function estimation by inverse optimization

- Elicited preferences for a prosumer: $\{\mathbf{x}^{\star k}, \boldsymbol{\lambda}^k\}_{k=1}^t$
- Parametric function: $\hat{v}(\mathbf{x}; \theta)$

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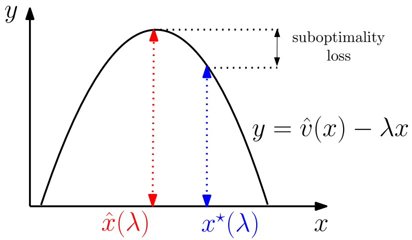
Minimize suboptimality loss

$$\min_{\theta} \sum_k \hat{v}(\hat{\mathbf{x}}^k; \theta) - \hat{v}(\mathbf{x}^{\star k}; \theta) + \langle \boldsymbol{\lambda}^k, \mathbf{x}^{\star k} - \hat{\mathbf{x}} \rangle$$

$$\hat{\mathbf{x}}^k \in \arg\max_{\mathbf{x}} \hat{v}(\mathbf{x}; \theta) - \langle \boldsymbol{\lambda}^k, \mathbf{x} \rangle$$



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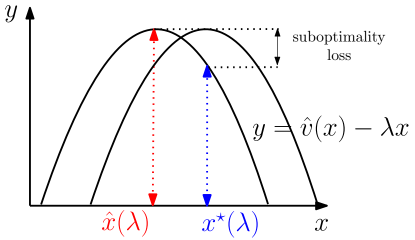

Machine learning-aided step size computation

Estimated dual problem

$$\hat{g}(\boldsymbol{\lambda}) = \max_{\mathbf{x} \in \mathcal{X}} \sum_{i=1}^{n} \hat{v}(\mathbf{x}_i) - \langle \boldsymbol{\lambda}, \mathbf{x}_i \rangle,$$

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ML-aided step size

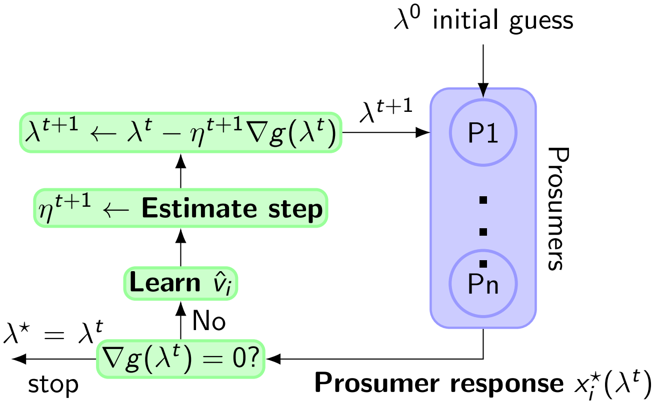
$$\hat{\eta}_t \in \arg\min_{\eta} \quad \hat{g}(\boldsymbol{\lambda}^t - \eta \nabla g(\boldsymbol{\lambda}^t))$$
$$\eta \in [\underline{\eta}^t, \bar{\eta}^t]$$

$$\boldsymbol{\lambda}^{t+1} \leftarrow \boldsymbol{\lambda}^t - \hat{\eta}_t \nabla g(\boldsymbol{\lambda}^t)$$

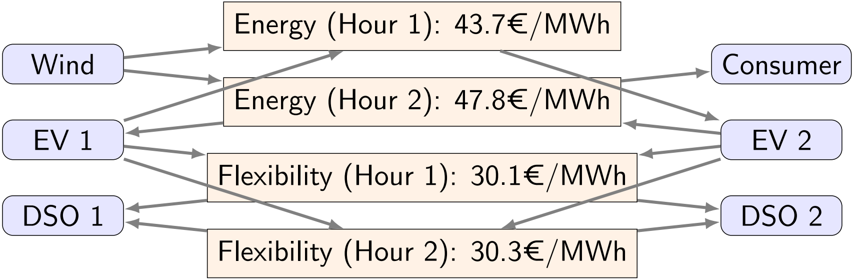
Pre-determined step size

$$\boldsymbol{\lambda}^{t+1} \leftarrow \boldsymbol{\lambda}^t - \eta_t \nabla g(\boldsymbol{\lambda}^t)$$

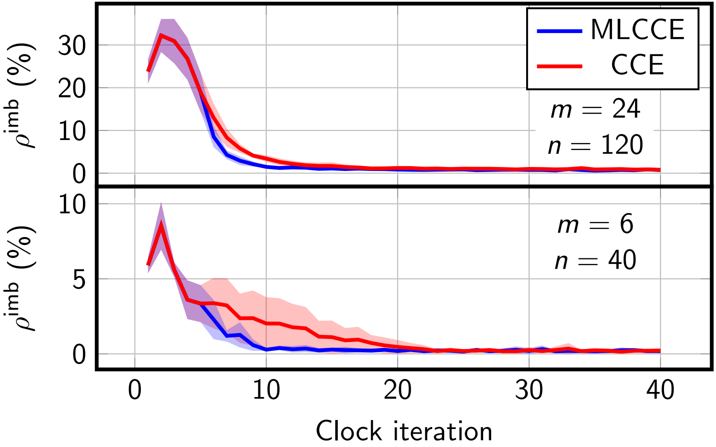
Machine learning-aided Combinatorial Clock Exchange



Example local energy market clearing

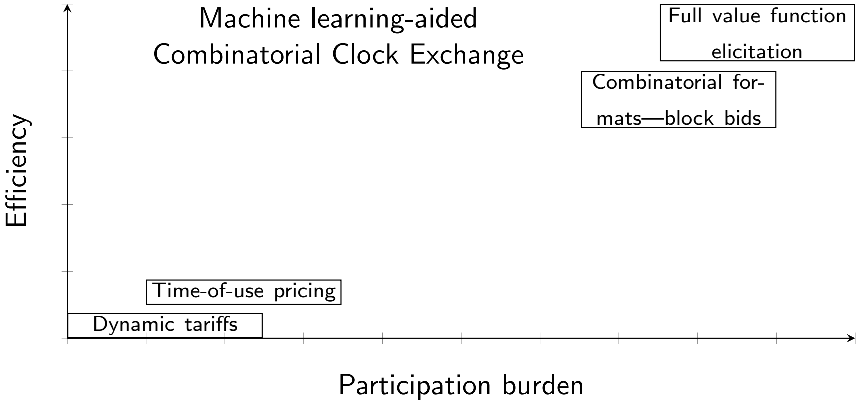


MLCCE needs fewer iterations than CCE



Note that the step size sequence in CCE has been pre-tuned.

Conclusion



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