Bounds for multi-horizon stochastic optimization with application to energy systems

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Advanced Decision-Making for Net-Zero Energy Systems

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In collaboration with:

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Güzin Bayraksan

Laureano Escudero

Giovanni Micheli

Asgeir Tomasgard

V Varagapriya

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Optimization Under Uncertainty in Energy Systems

- Real world decision problems in energy systems: **dynamic** and affected by **uncertainty**

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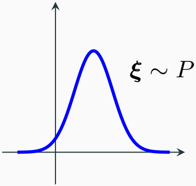
Deterministic Optimization

$$\min_{\mathbf{x}\in X} \quad f(\mathbf{x}, \xi)$$

ξ

Stochastic Optimization

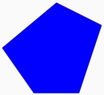
$$\min_{\mathbf{x}\in \mathcal{X}(\boldsymbol{\xi})} \quad \mathbb{E}_P\big[f(\mathbf{x}, \boldsymbol{\xi})\big]$$



$\boldsymbol{\xi} \sim P$

Robust Optimization

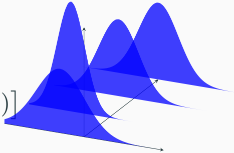
$$\min_{\mathbf{x}\in \mathcal{X}(\boldsymbol{\xi})} \max_{\boldsymbol{\xi}\in U} \quad f(\mathbf{x}, \boldsymbol{\xi})$$



$\boldsymbol{\xi} \in U$

Distributionally Robust Optimization

$$\min_{\mathbf{x}\in \mathcal{X}(\boldsymbol{\xi})} \max_{P\in \mathcal{P}} \quad \mathbb{E}_P\big[f(\mathbf{x}, \boldsymbol{\xi})\big]$$



$\boldsymbol{\xi} \sim P,\ P \in \mathcal{P}$

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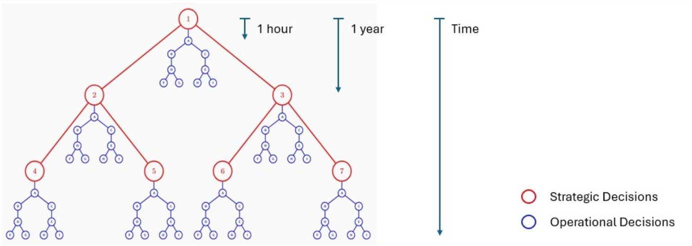
Uncertainty types

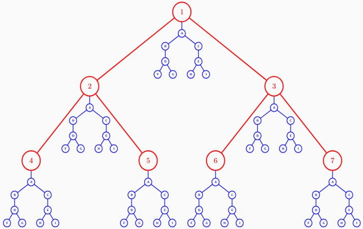
- **Strategic** examples
  - Investment cost
  - Carbon price
  - Fuel price development
- **Operational** examples
  - Load levels
  - Hydro reservoir inflow

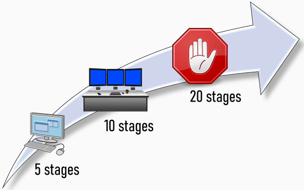


Kaut, Midthun, Werner, Tomasgard, Hellemo & Fodstad (2014). Multi-horizon stochastic programming. *Computational Management Science*, *11*, 179–193.

Multi-horizon Problems

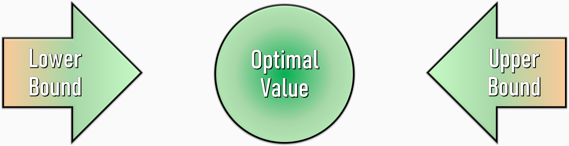
- **Multi-horizon optimization** forms a class of extremely **challenging problems**





Approximation techniques:

Lower and **upper bounds** to the optimal value for **multi-horizon optimization**



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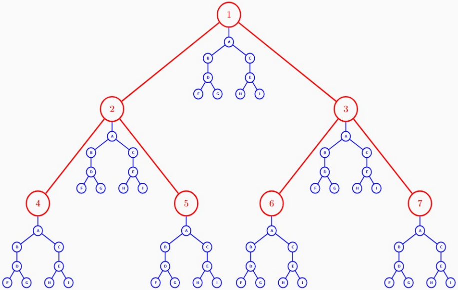
The Goal

Questions

1. How to bound **multi-horizon SO** via **expectation-based** approaches?



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The Goal

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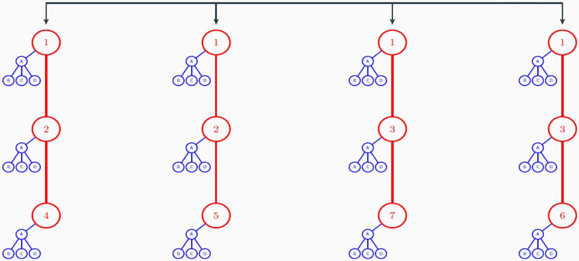
1. How to bound **multi-horizon SO** via **expectation-based** approaches?
2. How to bound **multi-horizon SO** via **relaxation** of non-anticipativities constraints?



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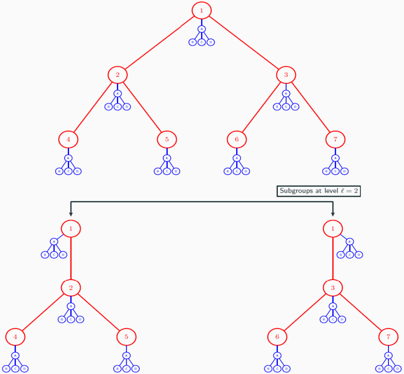
1. How to bound **multi-horizon SO** via **expectation-based** approaches?
2. How to bound **multi-horizon SO** via **relaxation** of non-anticipativities constraints?
3. How to bound **multi-horizon SO** via **scenario grouping**?



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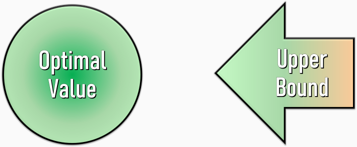
4. How to provide **feasible soluzions** to **multi-horizon SO** problems?



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Generation and Transmission Expansion Planning

Domestic Renewable Energy Sytstem Design

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Selected Literature Review for Multi-horizon Optimization

Methods

- Hellemo et al. (2012) and Kaut et al. (2014) first introduced **multi-horizon optimization models**;
- Werner et al. (2013) investigate the **time consistency property** in multi-horizon scenario trees;
- Vojvodic et al. (2023) and Jacobson et al. (2023) propose **Benders-type algorithms** to achieve near-optimal solutions;
- Zhang et al. (2024) integrate **Benders and Lagrangean decomposition methods** to solve multi-horizon LP problems;
- Rigaut et al. (2024) propose a **time-block decomposition scheme** using recursive Bellman-like equations.

Applications

- **Capacity expansion** planning of power systems (Seljom and Tomasgard (2015), Skar et al. (2016), Escudero and Monge (2018));
- Analysis of **multi-fuel energy markets** (Su et al. 2015);
- Operational planning of **pumped hydro power plants** (Abgottspon and Andersson (2016));
- Assessment of **policy actions in energy markets** (Seljom and Tomasgard (2017)).

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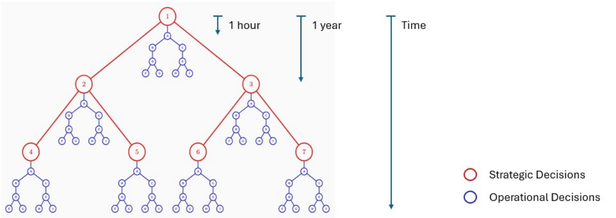
Multi Horizon Stochastic Program

MHSP

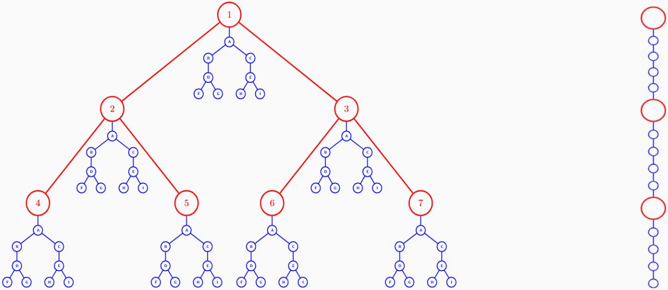
$$
\begin{aligned}
MHSP \quad := \quad & \min_{\mathbf{x},\mathbf{y}} \sum_{t=0}^{H} \sum_{n \in \mathcal{N}_t} \pi_n \Big(c_n x_n + \sum_{\omega \in \Omega_t} \pi_n^{\omega} \sum_{\tau \in \mathcal{T}_t} q_n^{\omega,\tau} y_n^{\omega,\tau} \Big) \\
\text{s.t.} \quad & A x_n = h_n, \qquad n \in \mathcal{N}_0, \\
& T_n x_{a(n)} + W_n x_n = h_n, \qquad n \in \mathcal{N}_t,\ t \in \mathcal{H} \setminus \{0\}, \\
& T_n^{\omega,1} x_n + W_n^{\omega,1} y_n^{\omega,1} = h_n^{\omega,1}, \qquad n \in \mathcal{N}_t,\ \omega \in \Omega_n,\ t \in \mathcal{H}, \\
& T_n^{\omega,\tau} y_n^{\omega,\tau-1} + W_n^{\omega,\tau} y_n^{\omega,\tau} = h_n^{\omega,\tau}, \qquad n \in \mathcal{N}_t,\ \tau \in \mathcal{T}_t \setminus \{1\},\ \omega \in \Omega_n,\ t \in \mathcal{H}, \\
& x_n \in \mathbb{R}_+^{n_t}, \qquad n \in \mathcal{N}_t, t \in \mathcal{H}, \\
& y_n^{\omega,\tau} \in \mathbb{R}_+^{n_t^{\tau}}, \qquad n \in \mathcal{N}_t,\ \tau \in \mathcal{T}_t,\ \omega \in \Omega_n,\ t \in \mathcal{H},
\end{aligned}
$$



Maggioni, Allevi, Tomasgard (2020) Bounds in multi-horizon stochastic programs, *Annals of Operations Research*, 292, 605—625.

Question 1

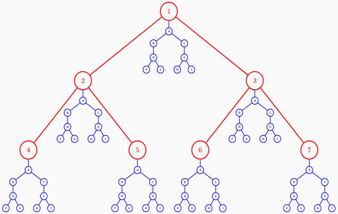
How to bound **multi-horizon SO** via **expectation-based** approaches?

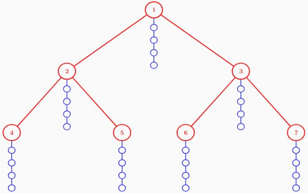


Multi-Horizon Operative Expected Value problem

$MHOEV \leq MHSP$

$$
\begin{aligned}
MHOEV \quad := \quad & \min_{\mathbf{x},\mathbf{y}} \sum_{t=0}^{H} \sum_{n\in\mathcal{N}_t} \pi_n (c_n x_n + \sum_{\tau\in\mathcal{T}_t} \bar{q}_n^{\tau} y_n^{\tau}) \\
\text{s.t.} \quad & A x_n = h_n, \qquad n \in \mathcal{N}_0, \\
& T_n x_{a(n)} + W_n x_n = h_n, \qquad n \in \mathcal{N}_t,\ t \in \mathcal{H}\setminus\{0\}, \\
& \bar{T}_n^1 x_n + \bar{W}_n^1 y_n^1 = \bar{h}_n^1, \qquad n \in \mathcal{N}_t,\ t \in \mathcal{H} \\
& \bar{T}_n^{\tau} y_n^{\tau-1} + \bar{W}_n^{\tau} y_n^{\tau} = \bar{h}_n^{\tau}, \qquad n \in \mathcal{N}_t,\ t \in \mathcal{H},\ \tau \in \mathcal{T}_t\setminus\{1\}.
\end{aligned}
$$





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Multi-Horizon Strategic Expected Value problem

$MHSEV \leq MHSP$

$$
\begin{aligned}
MHSEV \quad := \quad & \min_{\mathbf{x},\mathbf{y}} \sum_{t=0}^{H} \left(\bar{c}_t x_t + \sum_{\omega \in \Omega_t} \pi_t^{\omega} \sum_{\tau \in \mathcal{T}_t} q_t^{\omega,\tau} y_t^{\omega,\tau} \right) \\
\text{s.t.} \quad & A x_0 = h_0, \\
& \bar{T}_t x_{t-1} + \bar{W}_t x_t = \bar{h}_t, \ t \in \mathcal{H} \setminus \{0\}, \\
& T_t^{\omega,1} x_t + W_t^{\omega,1} y_t^{\omega,1} = h_t^{\omega,1}, \ \omega \in \Omega_t, \ t \in \mathcal{H}, \\
& T_t^{\omega,\tau} y_t^{\omega,\tau-1} + W_t^{\omega,\tau} y_t^{\omega,\tau} = h_t^{\omega,\tau}, \ \tau \in \mathcal{T}_t \setminus \{1\}, \ \omega \in \Omega_t, \ t \in \mathcal{H}.
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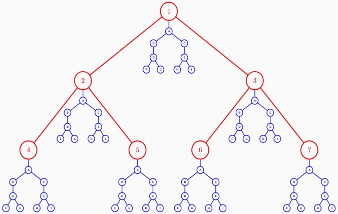

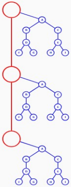

Micheli, G., Varagapriya, V., Maggioni, F. & Bayraksan, G. (2025) Bounds for multi-horizon stochastic optimization with application to power generation and transmission expansion planning. *In preparation*

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Multi-Horizon Expected Value

$MHEV \leq MHSP$

$$
\begin{aligned}
MHEV \quad := \quad & \min_{\mathbf{x},\mathbf{y}} \sum_{t=0}^{H} (\bar{c}_t x_t + \sum_{\tau \in \mathcal{T}_t} \bar{q}_t^\tau y_t^\tau) \\
\text{s.t.} \quad & A x_0 = \bar{h}_0, \\
& \bar{T}_t x_{t-1} + \bar{W}_t x_t = \bar{h}_t, \qquad t \in \mathcal{H} \setminus \{0\}, \\
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& \bar{T}_t^\tau y_t^{\tau-1} + \bar{W}_t^\tau y_t^\tau = \bar{h}_t^\tau, \qquad \tau \in \mathcal{T}_t \setminus \{1\}, \ t \in \mathcal{H}.
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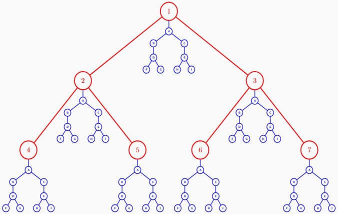

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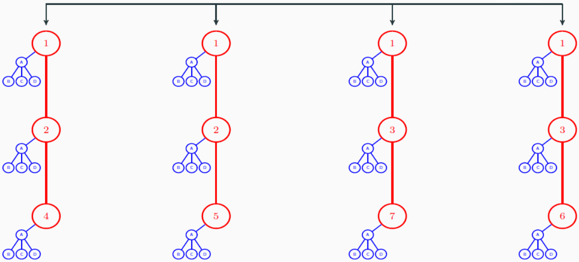
Question 2

How to bound **multi-horizon SO** via **relazation** of non-anticipativities constraints?



MHSWS

$$
\begin{aligned}
MHSWS \quad := \quad & \sum_{t=0}^{H} \sum_{\mathbf{s} \in \mathscr{S}} \pi^{\mathbf{s}} \min_{\mathbf{x},\mathbf{y}} \Big(c_t^{\mathbf{s}} x_t^{\mathbf{s}} + \sum_{\omega \in \Omega_t} \pi_{\mathbf{s},t}^{\omega} \sum_{\tau \in \mathcal{T}_t} q_{\mathbf{s},t}^{\omega,\tau} y_{\mathbf{s},t}^{\omega,\tau} \Big) \\
\text{s.t.} \qquad & A x_0 = h_0, \\
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& y_{\mathbf{s},t}^{\omega',\tau} = y_{\mathbf{s},t}^{\omega'',\tau},\ \forall \omega', \omega'' \in \Omega_t \text{ for which } \omega' = \omega'' \text{up to operational stage } \tau.
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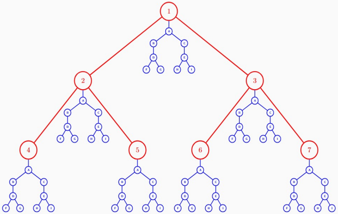

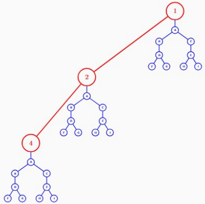

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Multi-Horizon Strategic Wait-and-See Program

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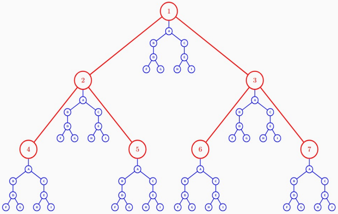

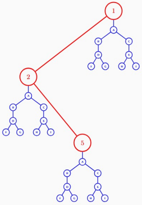

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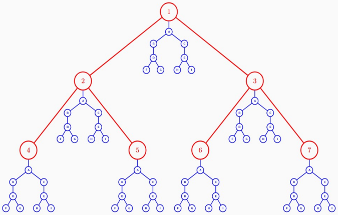

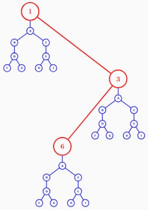

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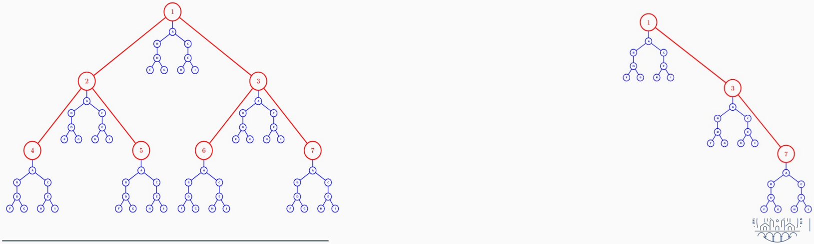

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Multi-Horizon Wait-and-See Program

MHWS

$$
\begin{aligned}
MHWS \quad := \quad & \sum_{t=0}^{H} \sum_{\mathbf{s}\in\mathscr{S}} \pi^{\mathbf{s}} \min_{\mathbf{x},\mathbf{y}} (c_t^{\mathbf{s}} x_t^{\mathbf{s}} + \sum_{\omega\in\Omega_t} \pi_{\mathbf{s},t}^{\omega} \sum_{\tau\in\mathcal{T}_t} q_{\mathbf{s},t}^{\omega,\tau} y_{\mathbf{s},t}^{\omega,\tau}) \\
\text{s.t.} \quad & A x_0 = h_0, \\
& T_t^{\mathbf{s}} x_{t-1}^{\mathbf{s}} + W_t^{\mathbf{s}} x_t^{\mathbf{s}} = h_t^{\mathbf{s}}, \qquad \mathbf{s}\in\mathscr{S},\ t\in\mathcal{H}\setminus\{0\}, \\
& T_{\mathbf{s},t}^{\omega,1} x_t^{\mathbf{s}} + W_{\mathbf{s},t}^{\omega,1} y_{\mathbf{s},t}^{\omega,1} = h_{\mathbf{s},t}^{\omega,1}, \qquad \mathbf{s}\in\mathscr{S}, \omega\in\Omega_t,\ t\in\mathcal{H}, \\
& T_{\mathbf{s},t}^{\omega,\tau} y_{\mathbf{s},t}^{\omega,\tau-1} + W_{\mathbf{s},t}^{\omega,\tau} y_{\mathbf{s},t}^{\omega,\tau} = h_{\mathbf{s},t}^{\omega,\tau}, \qquad \mathbf{s}\in\mathscr{S},\ \omega\in\Omega_t,\ \tau\in\mathcal{T}_t\setminus\{1\},\ t\in\mathcal{H}.
\end{aligned}
$$

Proposition

$$MHWS \le MHSWS \le MHSP.$$

If the objective function coefficients and recourse matrix are fixed then:

$$MHEV \le MHWS$$

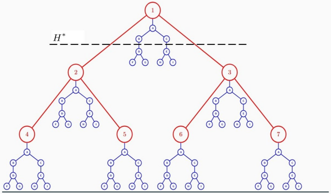
$$MHEV \le MHOEV.$$

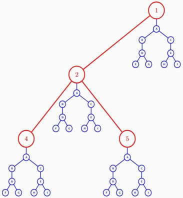
Maggioni, Allevi, Tomasgard (2020) Bounds in multi-horizon stochastic programs, *Annals of Operations Research*, 292, 605—625.

$MHC(H^*)$

$$MHC(H^*) := \min_{\mathbf{x},\mathbf{y}} \sum_{t \in \mathcal{H}} \sum_{\mathbf{s} \in \mathcal{S}} \pi^{\mathbf{s}} \left(c_t^{\mathbf{s}} x_t^{\mathbf{s}} + \sum_{\omega \in \Omega_t} \pi_{\mathbf{s},t}^{\omega} \sum_{\tau \in \mathcal{T}_t} q_{\mathbf{s},t}^{\omega,\tau} y_{\mathbf{s},t}^{\omega,\tau} \right)$$

$$\begin{aligned}
\text{s.t.} \quad & A x_0 = h_0, \\
& T_t^{\mathbf{s}} x_{t-1}^{\mathbf{s}} + W_t^{\mathbf{s}} x_t^{\mathbf{s}} = h_t^{\mathbf{s}}, \ \mathbf{s} \in \mathcal{S}, \ t \in \mathcal{H} \setminus \{0\}, \\
& T_{\mathbf{s},t}^{\omega,1} x_t^{\mathbf{s}} + W_{\mathbf{s},t}^{\omega,1} y_{\mathbf{s},t}^{\omega,1} = h_{\mathbf{s},t}^{\omega,1}, \ \mathbf{s} \in \mathcal{S}, \ \omega \in \Omega_t, \ t \in \mathcal{H}, \\
& T_{\mathbf{s},t}^{\omega,\tau} y_{\mathbf{s},t}^{\omega,\tau-1} + W_{\mathbf{s},t}^{\omega,\tau} y_{\mathbf{s},t}^{\omega,\tau} = h_{\mathbf{s},t}^{\omega,\tau}, \ \mathbf{s} \in \mathcal{S}, \ \omega \in \Omega_t, \ \tau \in \mathcal{T}_t \setminus \{1\}, t \in \mathcal{H}, \\
& x_t^{\mathbf{s}'} = x_t^{\mathbf{s}''}, \ \forall \mathbf{s}', \mathbf{s}'' \in \mathcal{S}, \ \text{for which } \mathbf{s}' = \mathbf{s}'' \text{up to strategic stage } t, \ t \geq H^*, \\
& y_{\mathbf{s},t}^{\omega',\tau} = y_{\mathbf{s},t}^{\omega'',\tau}, \ \forall \omega', \omega'' \in \Omega_t, \ \text{for which } \omega' = \omega'' \text{up to operational stage } \tau.
\end{aligned}$$



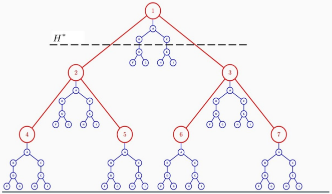


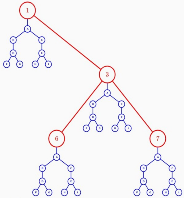
Micheli, G., Escudero, L., Maggioni, F. & Bayraksan, G. (2025). Multi-horizon optimization for domestic renewable energy system design under uncertainty. *Under Review*, https://doi.org/10.48550/arXiv.2505.15167.

$MHC(H^*)$

$$MHC(H^*) := \min_{\mathbf{x},\mathbf{y}} \sum_{t\in\mathcal{H}} \sum_{\mathbf{s}\in\mathcal{S}} \pi^{\mathbf{s}} \Big(c_t^{\mathbf{s}} x_t^{\mathbf{s}} + \sum_{\omega\in\Omega_t} \pi_{\mathbf{s},t}^{\omega} \sum_{\tau\in\mathcal{T}_t} q_{\mathbf{s},t}^{\omega,\tau} y_{\mathbf{s},t}^{\omega,\tau} \Big)$$

$$\begin{aligned}
\text{s.t.}\quad & Ax_0 = h_0, \\
& T_t^{\mathbf{s}} x_{t-1}^{\mathbf{s}} + W_t^{\mathbf{s}} x_t^{\mathbf{s}} = h_t^{\mathbf{s}},\ \mathbf{s}\in\mathcal{S},\ t\in\mathcal{H}\setminus\{0\}, \\
& T_{\mathbf{s},t}^{\omega,1} x_t^{\mathbf{s}} + W_{\mathbf{s},t}^{\omega,1} y_{\mathbf{s},t}^{\omega,1} = h_{\mathbf{s},t}^{\omega,1},\ \mathbf{s}\in\mathcal{S},\ \omega\in\Omega_t,\ t\in\mathcal{H}, \\
& T_{\mathbf{s},t}^{\omega,\tau} y_{\mathbf{s},t}^{\omega,\tau-1} + W_{\mathbf{s},t}^{\omega,\tau} y_{\mathbf{s},t}^{\omega,\tau} = h_{\mathbf{s},t}^{\omega,\tau},\ \mathbf{s}\in\mathcal{S},\ \omega\in\Omega_t,\ \tau\in\mathcal{T}_t\setminus\{1\}, t\in\mathcal{H}, \\
& x_t^{\mathbf{s}'} = x_t^{\mathbf{s}''},\ \forall \mathbf{s}',\mathbf{s}''\in\mathcal{S},\ \text{for which } \mathbf{s}'=\mathbf{s}''\text{up to strategic stage } t,\ t\geq H^*, \\
& y_{\mathbf{s},t}^{\omega',\tau} = y_{\mathbf{s},t}^{\omega'',\tau},\ \forall\omega',\omega''\in\Omega_t,\ \text{for which } \omega'=\omega''\text{up to operational stage } \tau.
\end{aligned}$$





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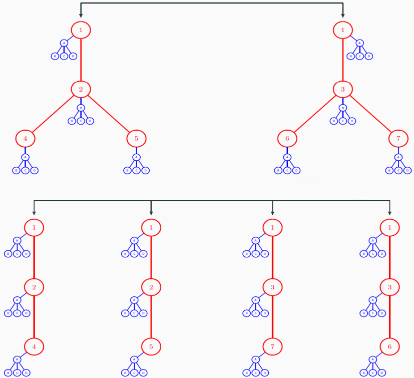
Micheli, G., Escudero, L., Maggioni, F. & Bayraksan, G. (2025). Multi-horizon optimization for domestic renewable energy system design under uncertainty. *Under Review*, `https://doi.org/10.48550/arXiv.2505.15167`.

Question 3

How to bound **multi-horizon SO** via **scenario grouping**?



Lower Bounds for Multi-Horizon Stochastic Optimization

Assumption: the random process has finitely many possible realizations.

Multi-Horizon SO Problem

The starting problem is split into n subproblems, independently solved.

$\swarrow$ $\Downarrow$ $\Downarrow$ $\searrow$

Subproblem 1	**Subproblem** 2	$\ldots$	**Subproblem** n

Optimal values of the subproblems are combined to obtain a:

Lower Bound
to the original problem

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Assumption: the random process has finitely many possible realizations.

Multi-Horizon SO Problem

The starting problem is split into n subproblems, independently solved.

$\swarrow$ $\Downarrow$ $\Downarrow$ $\searrow$

Subproblem 1	**Subproblem** 2	$\dots$	**Subproblem** n

Optimal values of the subproblems are combined to obtain a:

Lower Bound
to the original problem

Assumption: the random process has finitely many possible realizations.

Multi-Horizon SO Problem

The starting problem is split into n subproblems, independently solved.

$\Swarrow$ $\Downarrow$ $\Downarrow$ $\Searrow$

Subproblem 1	**Subproblem** 2	$\ldots$	**Subproblem** n

Optimal values of the subproblems are combined to obtain a:

Lower Bound
to the original problem

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Multi-horizon SO Problem

strategic uncertainty: (Ξ, Π)

operational uncertainty at strategic node n: (Ω_n, P_n)

$\swarrow \qquad \Downarrow \qquad \Downarrow \qquad \searrow$

Subproblem 1	**Subproblem** 2		**Subproblem** m_ℓ
$\left(\Xi_1^{(\ell)}, \Pi_1^{(\ell)}\right)$	$\left(\Xi_2^{(\ell)}, \Pi_2^{(\ell)}\right)$	$\dots$	$\left(\Xi_{m_l}^{(\ell)}, \Pi_{m_\ell}^{(\ell)}\right)$

Refinement Chain

$$\begin{array}{cc}
\Xi & \Pi \\
(\Xi_1^{(\ell)}, \dots, \Xi_{m_\ell}^{(\ell)}) & (\Pi_1^{(\ell)}, \dots \Pi_{m_\ell}^{(\ell)}) \\
\vdots & \vdots \\
(\Xi_1^{(2)}, \dots, \Xi_{m_2}^{(2)}) & (\Pi_1^{(2)}, \dots, \Pi_{m_2}^{(2)}) \\
(\{\omega_1\}, \dots, \{\omega_N\}) & (\Pi_1^{(1)} = \delta_{\omega_1}, \dots, \Pi_N^{(1)} = \delta_{\omega_N})
\end{array}$$

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Dissection of the Strategic Probability Measure

Multi-horizon SO Problem

strategic uncertainty: (Ξ, Π)

operational uncertainty at strategic node n: (Ω_n, P_n)

$\swarrow \quad \Downarrow \quad \Downarrow \quad \searrow$

Subproblem 1	**Subproblem** 2		**Subproblem** m_ℓ
$\left(\Xi_1^{(\ell)}, \Pi_1^{(\ell)}\right)$	$\left(\Xi_2^{(\ell)}, \Pi_2^{(\ell)}\right)$	$\dots$	$\left(\Xi_{m_l}^{(\ell)}, \Pi_{m_\ell}^{(\ell)}\right)$

Support:

(i) $\Xi_i^{(\ell)} = \cup_{\Xi_k^{(\ell-1)} \subseteq \Xi_i^{(\ell)}} \Xi_k^{(\ell-1)}$

(ii) $\bigcap_{i=1}^{m_\ell} \Xi_i^{(\ell)} = \Xi_f$ for each strategic level ℓ ($f = 0$ disjoint subsets).

Probabilities:

(iii) $\Pi = \sum_{i=1}^{m_\ell} \phi_i^{(\ell)} \Pi_i^{(\ell)}$, with $\sum_{i=1}^{m_\ell} \phi_i^{(\ell)} = 1$, $\phi_i^{(\ell)} \geq 0$, $i = 1, \dots, m_\ell$

(iv) each $\Pi_i^{(\ell)}$ can be written as a **convex combination of** $\left\{\Pi_i^{(\ell-1)}\right\}$

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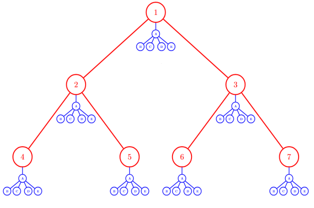
Multi-horizon SO Problem

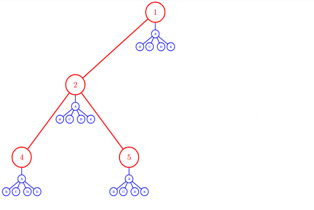
strategic uncertainty: (Ξ, Π)

operational uncertainty at strategic node n: (Ω_n, P_n)

$\Swarrow$ $\Downarrow$ $\Downarrow$ $\Searrow$

Subproblem 1	**Subproblem** 2		**Subproblem** m_ℓ
$\left(\Xi_1^{(\ell)}, \Pi_1^{(\ell)}\right)$	$\left(\Xi_2^{(\ell)}, \Pi_2^{(\ell)}\right)$	$\dots$	$\left(\Xi_{m_l}^{(\ell)}, \Pi_{m_\ell}^{(\ell)}\right)$





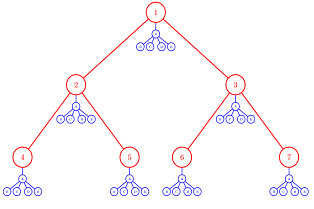
Multi-horizon SO Problem

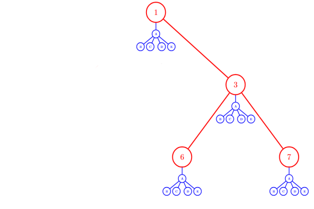
strategic uncertainty: (Ξ, Π)

operational uncertainty at strategic node n: (Ω_n, P_n)

$\swarrow \quad \Downarrow \quad \Downarrow \quad \searrow$

Subproblem 1	**Subproblem** 2	$\ldots$	**Subproblem** m_ℓ
$\left(\Xi_1^{(\ell)}, \Pi_1^{(\ell)}\right)$	$\left(\Xi_2^{(\ell)}, \Pi_2^{(\ell)}\right)$		$\left(\Xi_{m_l}^{(\ell)}, \Pi_{m_\ell}^{(\ell)}\right)$





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Multi-horizon SO Problem

strategic uncertainty: (Ξ, Π)

operational uncertainty at strategic node n: (Ω_n, P_n)

$\swarrow \quad \Downarrow \quad \Downarrow \quad \searrow$

Subproblem 1	**Subproblem** 2		**Subproblem** m_ℓ
$\left(\Omega^{(\gamma)}_{n,1}, P^{(\gamma)}_{n,1}\right)$	$\left(\Omega^{(\gamma)}_{n,2}, P^{(\gamma)}_{n,2}\right)$	$\ldots$	$\left(\Omega^{(\gamma)}_{n,m_\gamma}, P^{(\gamma)}_{n,m_\gamma}\right)$

Supports:

(i) $\Omega^{(\gamma)}_{n,j} = \cup_{\Omega^{(\gamma-1)}_{n,j} \subseteq \Omega^{(\gamma)}_{n,j}} \Omega^{(\gamma-1)}_{n,j}$

(ii) $\bigcap_{j=1}^{m_\gamma} \Omega^{(\gamma)}_{n,j} = \Omega_f$ for each operational level γ ($f = 0$ disjoint subsets).

Probabilities:

(iii) $P_n = \sum_{j=1}^{m_\gamma} \phi^{(\gamma)}_{n,j} P^{(\gamma)}_{n,j}$, with $\sum_{j=1}^{m_\gamma} \phi^{(\gamma)}_{n,j} = 1$, $\phi^{(\gamma)}_{n,j} \geq 0$

(iv) each $P^{(\gamma)}_{n,j}$ can be written as a **convex combination of** $\left\{P^{(\gamma-1)}_{n,j}\right\}$

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Dissection of the Operational Probability Measure

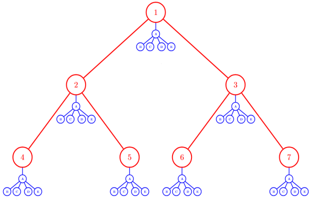
Multi-horizon SO Problem

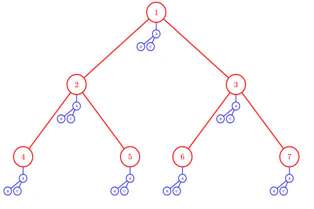
strategic uncertainty: (Ξ, Π)

operational uncertainty at strategic node n: $(\Omega_n,\ P_n)$

$\swarrow \qquad \Downarrow \qquad \Downarrow \qquad \searrow$

Subproblem 1	**Subproblem** 2		**Subproblem** m_ℓ
$\left(\Omega^{(\gamma)}_{n,1}, P^{(\gamma)}_{n,1}\right)$	$\left(\Omega^{(\gamma)}_{n,2}, P^{(\gamma)}_{n,2}\right)$	$\dots$	$\left(\Omega^{(\gamma)}_{n,m_\gamma}, P^{(\gamma)}_{n,m_\gamma}\right)$





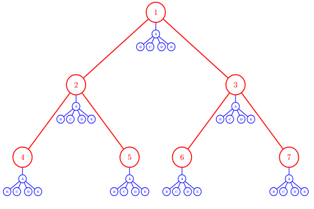
Multi-horizon SO Problem

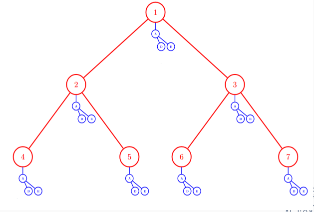
strategic uncertainty: (Ξ, Π)

operational uncertainty at strategic node n: $(\Omega_n,\ P_n)$

$\swarrow \qquad \Downarrow \qquad \Downarrow \qquad \searrow$

Subproblem 1	**Subproblem** 2		**Subproblem** m_ℓ
$\left(\Omega^{(\gamma)}_{n,1}, P^{(\gamma)}_{n,1}\right)$	$\left(\Omega^{(\gamma)}_{n,2}, P^{(\gamma)}_{n,2}\right)$	$\dots$	$\left(\Omega^{(\gamma)}_{n,m_\gamma}, P^{(\gamma)}_{n,m_\gamma}\right)$





Dissection of Strategic/Operational Probability Measures

Definition (Multi-Horizon Nodal Group / Multi-Horizon Expected Group)

$$MHNG(\ell,\gamma) := \sum_{i=1}^{m_\ell} \phi_i^{(\ell)} \Big(\sum_{t\in\mathcal{H}} \sum_{n\in\Xi_i^{(\ell)}\cap\mathcal{N}_t} \sum_{j=1}^{m_\gamma} \phi_{n,j}^{(\gamma)} \Pi_{i,n}^{(\ell)} (c_n \hat{x}_n + \sum_{\omega\in\Omega_{n,j}^{(\ell,\gamma)}} P_{n,\omega}^{(\gamma)} \sum_{\tau\in\mathcal{T}_t} q_n^{\omega,\tau} \hat{y}_n^{\omega,\tau}) \Big)$$

$$MHEG(\ell,\gamma) := \sum_{i=1}^{m_\ell} \phi_i^{(\ell)} \sum_{j=1}^{M_\gamma} \phi_{\mathsf{OG},j}^{(\gamma)} MHG(\Xi_i^{(\ell)}, \Omega_{\mathsf{OG},j}^{(\gamma)}) \quad [\text{with } \phi_{\mathsf{OG},j}^{(\gamma)} = \prod_{n\in\mathcal{N}_t, t\in\mathcal{H}} \phi_{n,j}^{(\gamma)}]$$

Proposition

For a fixed operational level γ:

(a) $MHNG(1,\gamma) \leq MHNG(2,\gamma) \leq \ldots \leq MHNG(\ell,\gamma) \leq \ldots \leq MHSP$

(b) $MHEG(1,\gamma) \leq MHEG(2,\gamma) \leq \ldots \leq MHEG(\ell,\gamma) \leq \ldots \leq MHSP$

For a fixed strategic level ℓ:

(b) $MHNG(\ell,1) \leq MHNG(\ell,2) \leq \ldots \leq MHNG(\ell,\gamma) \leq \ldots \leq MHSP$

(d) $MHEG(\ell,1) \leq MHEG(\ell,2) \leq \ldots \leq MHEG(\ell,\gamma) \leq \ldots \leq MHSP$

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Micheli, V. Varagapriya, F. Maggioni & G. Bayraksan (2025) Bounds for multi-horizon stochastic optimization with application to power generation and transmission expansion planning. *In preparation*

Dissection of Strategic/Operational Probability Measures

Definition (Multi-Horizon Nodal Group / Multi-Horizon Expected Group)

$$MHNG(\ell,\gamma) := \sum_{i=1}^{m_\ell} \phi_i^{(\ell)} \Big(\sum_{t\in\mathcal{H}} \sum_{n\in\Xi_i^{(\ell)}\cap\mathcal{N}_t} \sum_{j=1}^{m_\gamma} \phi_{n,j}^{(\gamma)} \Pi_{i,n}^{(\ell)} (c_n \hat{x}_n + \sum_{\omega\in\Omega_{n,j}^{(\ell,\gamma)}} P_{n,\omega}^{(\gamma)} \sum_{\tau\in\mathcal{T}_t} q_n^{\omega,\tau} \hat{y}_n^{\omega,\tau}) \Big)$$

$$MHEG(\ell,\gamma) := \sum_{i=1}^{m_\ell} \phi_i^{(\ell)} \sum_{j=1}^{M_\gamma} \phi_{\mathsf{OG},j}^{(\gamma)} MHG(\Xi_i^{(\ell)}, \Omega_{\mathsf{OG},j}^{(\gamma)}) \quad [\text{with } \phi_{\mathsf{OG},j}^{(\gamma)} = \prod_{n\in\mathcal{N}_t, t\in\mathcal{H}} \phi_{n,j}^{(\gamma)}]$$

Proposition

For a fixed operational level γ:

(a) $MHNG(1,\gamma) \leq MHNG(2,\gamma) \leq \ldots \leq MHNG(\ell,\gamma) \leq \ldots \leq MHSP$

(b) $MHEG(1,\gamma) \leq MHEG(2,\gamma) \leq \ldots \leq MHEG(\ell,\gamma) \leq \ldots \leq MHSP$

For a fixed strategic level ℓ:

(b) $MHNG(\ell,1) \leq MHNG(\ell,2) \leq \ldots \leq MHNG(\ell,\gamma) \leq \ldots \leq MHSP$

(d) $MHEG(\ell,1) \leq MHEG(\ell,2) \leq \ldots \leq MHEG(\ell,\gamma) \leq \ldots \leq MHSP$

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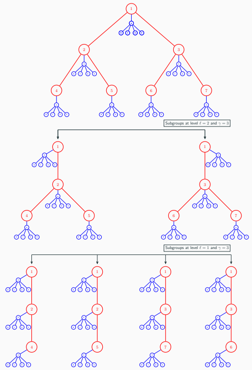
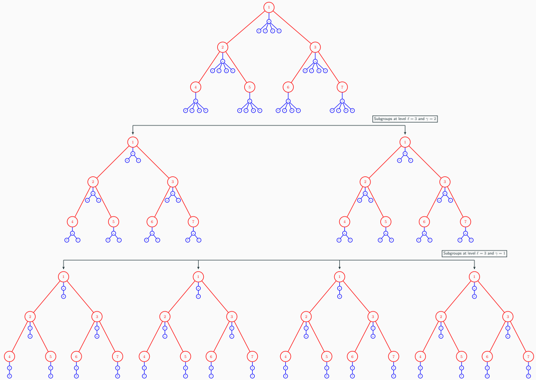
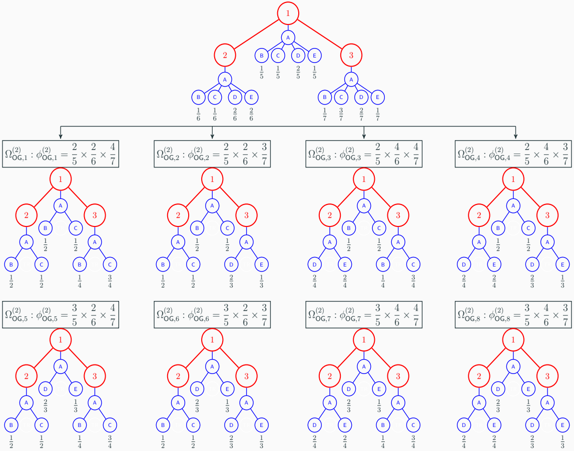
Examples of nodal strategic and operational chains $MHNG(\ell, \gamma)$





Examples of expectation-based operational groups $MHEG(\ell, \gamma)$



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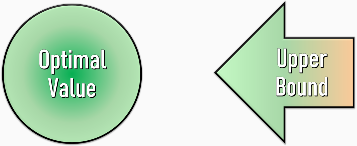
Question 4

How to provide feasible soluzions to **multi-horizon SO** problems?

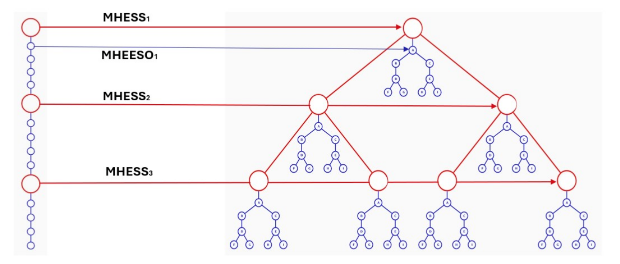


Upper Bounds

Proposition

The following chains of inequalities hold true:

- $MHSP \leq MHEES_1 \leq MHEES_2 \leq \ldots \leq MHEES_{H-1}$;
- $MHSP \leq MHEESO_1^{\tau} \leq MHEESO_2^{\tau} \leq \ldots \leq MHEESO_{H-1}^{\tau}, \tau \in \mathcal{T}_t$;
- $MHEES_t \leq MHEESO_t^{\tau}$, $\tau \in \mathcal{T}_t$, $t \in \mathcal{H} \setminus \{H\}$.



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Maggioni, Allevi, Tomasgard (2020) Bounds in multi-horizon stochastic programs, *Annals of Operations Research*, 292, 605—625.

A rolling horizon approach to compute upper bounds

Algorithm 1: Algorithm to compute a feasible solution to the MHSP

1: Input: $\hat{e}$, $\hat{e}^R$, ϕ_e
2: **for** $\kappa = 1$ to $|\mathcal{H}| - \hat{e} + 1$ **do**
3:   **for** $r \in \mathcal{N}_\kappa$ **do**
4:     Construct set $\overline{\mathcal{S}}^r := \{n \in \mathcal{S}^r : h(n) < \kappa + \hat{e}\}$
5:     **for** $e' = \kappa + \hat{e}$ to $\min(\kappa + \hat{e} + \hat{e}^R - 1, |\mathcal{H}|)$ **do**
6:       Construct set $\tilde{\mathcal{N}}^r_{e'}$ by randomly selecting with probability $\phi_{e'}$ strategic nodes $n' \in \mathcal{S}^r \cap \mathcal{N}_{e'}$, with $a(n') \in \overline{\mathcal{S}}^r \cup \tilde{\mathcal{N}}^r_{e'-1}$
7:     **end for**
8:     $\overline{\overline{\mathcal{S}}}^r := \bigcup_{e'=\kappa+\hat{e}}^{\min(\kappa+\hat{e}+\hat{e}^R-1,|\mathcal{H}|)} \tilde{\mathcal{N}}^r_{e'}$
9:     Define set $\overline{\mathcal{N}}^r := r \cup \overline{\mathcal{S}}^r \cup \overline{\overline{\mathcal{S}}}^r$
10:     Solve the r-submodel, defined as a replica of the MHSP model restricted to set $\overline{\mathcal{N}}^r$
11:     Fix variables x_r and $y_r^{\omega,\tau}$, $\omega \in \Omega_r, \tau \in \mathcal{T}_t$, to the solution of the r-submodel
12:     **if** $\kappa = |\mathcal{H}| - \hat{e} + 1$ **then**
13:       Fix variables x_n and $y_n^{\omega,\tau}$, $\omega \in \Omega_n, \tau \in \mathcal{T}_t$ to the solution of the r-submodel for any $n \in \overline{\mathcal{S}}^r$
14:     **end if**
15:   **end for**
16: **end for**
17: Output: A feasible solution $(x_n, y_n^{\omega,\tau})_{\omega\in\Omega_n,\tau\in\mathcal{T}_t,n\in\mathcal{N}_t,t\in\mathcal{H}}$.

Input Parameters

- $\hat{e}$: Number of non–relaxed stages
- $\hat{e}^R$: Number of relaxed stages
- ϕ_e: Selection probability (in relaxed stages)

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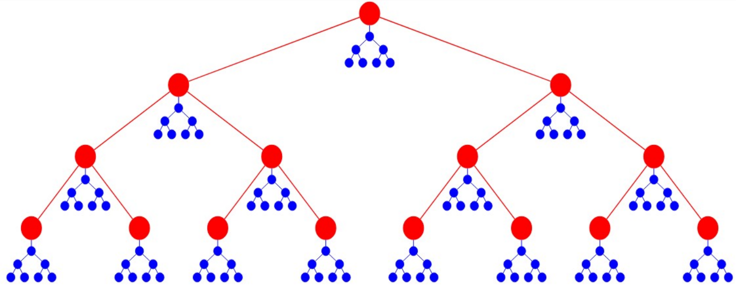
Micheli, G., Escudero, L., Maggioni, F. & Bayraksan, G. (2025) Multi-horizon optimization for domestic renewable energy system design under uncertainty. *Under review*, https://doi.org/10.48550/arXiv.2505.15167.

A rolling horizon approach to compute upper bounds

Input Parameters

- $\hat{e} = 2$: Number of non–relaxed stages
- $\hat{e}^R = 1$: Number of relaxed stages
- $\phi_e = \frac{1}{2}$: Selection probability (in relaxed stages)



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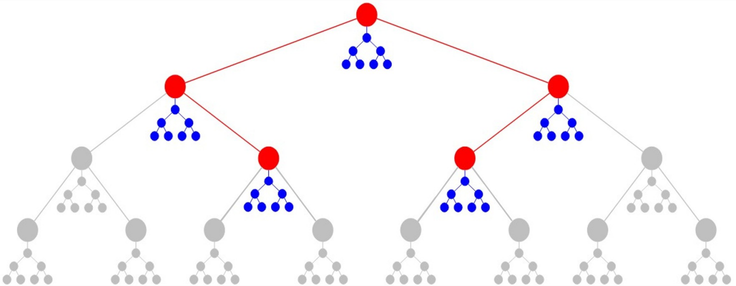

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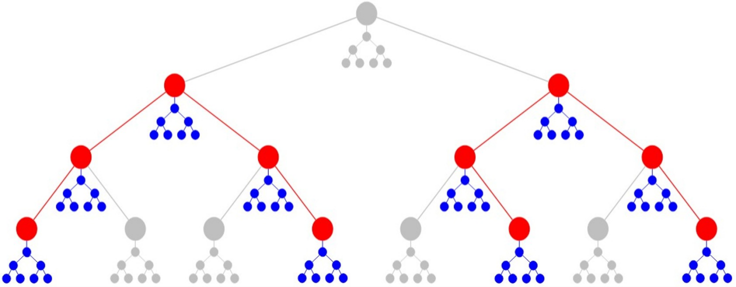

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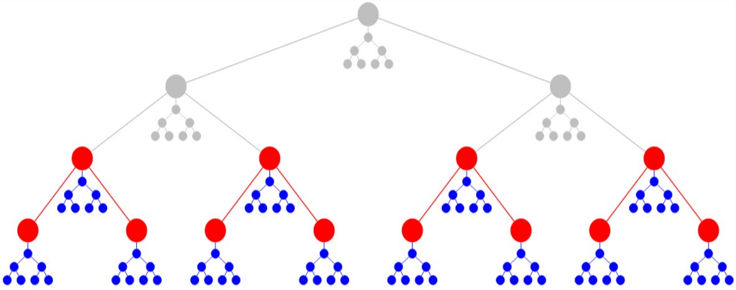

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- The problem consists in planning investments in **generation**, **transmission** and **storage** facilities, in order to supply at minimum cost the electricity demand, while achieving some decarbonization targets.
- **Decision variables**:
  1. **strategic**: The new capacity for generation, transmission and storage facilities in the long-term;
  2. **operational**: How to operate the power system to supply the electricity demand in every hour of representative days.
- **Uncertain parameters**:
  1. **Strategic**: investment costs and fuel prices;
  2. **Operational**: solar radiation, wind speed and natural inflows.

Problem Formulation

- The decision maker (e.g., the Independent System Operator, the Authority, or the regulator) seeks at the minimization of total system costs:

Objective Function

$$\min_{\mathbf{f},\mathbf{v},\mathbf{u},\mathbf{d},\mathbf{p},\mathbf{w}} \sum_{t=0}^{H} \sum_{n\in\mathcal{N}_t} \pi_n \Bigg(\sum_{l\in\mathcal{L}} C_{n,l}\ f_{n,l} + \sum_{k\in\mathcal{K}} \hat{C}_{n,k}\ v_{n,k} + \sum_{b\in\mathcal{B}} \tilde{C}_{n,b}\ u_{n,b} + \\ + ND_t \sum_{\omega\in\Omega_t} \pi_n^{\omega} \sum_{\tau\in\mathcal{T}_t} \Big(\sum_{k\in\mathcal{K}} \hat{c}_{n,k}\ p_{n,k}^{\omega,\tau} + \sum_{b\in\mathcal{B}} \tilde{c}_{t,b}\ w_{n,b,-}^{\omega,\tau} \Big) \Bigg)$$

- Constraints of the problem are divided into 3 groups:
    1. **Investment** constraints;
    2. **Operational** constraints;
    3. **Policy** constraints.

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1. Investment constraints

Investment constraints control the **strategic** investment decisions by:

- Defining the total capacity as the sum of the existing and the new capacity;
- Setting upper bounds to the installed capacity at the horizon.

Investment Model

$$F_{0,l} = \hat{F}_l + f_{0,l}, \quad l \in \mathcal{L},$$

$$F_{n,l} = F_{a(n),l} + f_{n,l}, \quad l \in \mathcal{L},\ n \in \mathcal{N} \setminus \{0\},$$

$$F_{n,l} \leq \overline{F}_l, \quad l \in \mathcal{L},\ n \in \mathcal{N}_H,$$

$$V_{0,k} = \hat{V}_k + v_{0,k}, \quad k \in \mathcal{K},$$

$$V_{n,k} = V_{a(n),k} + v_{n,k}, \quad k \in \mathcal{K},\ n \in \mathcal{N} \setminus \{0\},$$

$$V_{n,k} \leq \overline{V}_k, \quad k \in \mathcal{K},\ n \in \mathcal{N}_H,$$

$$U_{0,b} = \hat{U}_b + u_{0,b}, \quad b \in \mathcal{B},$$

$$U_{n,b} = U_{a(n),b} + u_{n,b}, \quad b \in \mathcal{B},\ n \in \mathcal{N} \setminus \{0\},$$

$$U_{n,b} \leq \overline{U}_b, \quad b \in \mathcal{B},\ n \in \mathcal{N}_H.$$

Operational constraints control the hourly **operations** in representative days:

- Ensuring the load supply;
- Imposing upper bounds to the operational decision variables;
- Imposing energy balances and minimum energy levels for storage facilities.

Operational Model

$$\sum_{k\in\mathcal{K}_z} p^{\omega,\tau}_{n,k} + \sum_{b\in\mathcal{B}_z} w^{\omega,\tau}_{n,b,-} + \sum_{l\in BS_z} d^{\omega,\tau}_{n,l} = D^{\tau}_{t,z} + \sum_{b\in\mathcal{B}_z} w^{\omega,\tau}_{n,b,+} + \sum_{l\in FS_z} d^{\omega,\tau}_{n,l},$$

$$z\in\mathcal{Z},\ n\in\mathcal{N}_t,\ \tau\in\mathcal{T}_t,\ \omega\in\Omega_t,\ t\in\mathcal{H},$$

$$d^{\omega,\tau}_{n,l} \le F_{n,l},\quad l\in\mathcal{L}, n\in\mathcal{N}_t,\ \tau\in\mathcal{T}_t,\ \omega\in\Omega_t,\ t\in\mathcal{H},$$

$$p^{\omega,\tau}_{n,k} \le R^{\omega,\tau}_{n,k} V_{n,k},\quad k\in\mathcal{K},\ n\in\mathcal{N}_t,\ \tau\in\mathcal{T}_t,\ \omega\in\Omega_t,\ t\in\mathcal{H},$$

$$w^{\omega,\tau}_{n,b} \le E_b U_{n,b},\quad b\in\mathcal{B},\ n\in\mathcal{N}_t,\ \tau\in\mathcal{T}_t,\ \omega\in\Omega_t,\ t\in\mathcal{H},$$

$$w^{\omega,\tau}_{n,b,-} \le U_{n,b},\quad b\in\mathcal{B},\ n\in\mathcal{N}_t,\ \tau\in\mathcal{T}_t,\ \omega\in\Omega_t,\ t\in\mathcal{H},$$

$$w^{\omega,\tau}_{n,b,+} \le U_{n,b},\quad b\in\mathcal{B},\ n\in\mathcal{N}_t,\ \tau\in\mathcal{T}_t,\ \omega\in\Omega_t,\ t\in\mathcal{H},$$

$$w^{\omega,\tau}_{n,b} = w^{\omega,\tau-1}_{n,b} + \lambda^+_b w^{\omega,\tau}_{n,b,+} - \lambda^-_b w^{\omega,\tau}_{n,b,-} + I^{\omega,\tau}_{n,b},\ b\in\mathcal{B}, n\in\mathcal{N}_t, \tau\in\mathcal{T}_t,\ \omega\in\Omega_n, t\in\mathcal{H},$$

$$w^{\omega,O_t}_{n,b} \ge \hat{w}^F_b,\quad b\in\mathcal{B},\ \omega\in\Omega_n,\ n\in\mathcal{N}_t,\ t\in\mathcal{H}.$$

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3. Policy constraints

Policy constraints promote the transition to **decarbonized** power systems by:

- Imposing maximum daily CO_2 emissions;
- Ensuring a minimum share of the total generation capacity that must be allocated to non-programmable renewable power plants.

Policy objectives

$$\sum_{k \in \mathcal{K}} \sum_{\tau \in \mathcal{T}_t} M_k p_{n,k}^{\omega,\tau} \leq \overline{M}_t, \quad n \in \mathcal{N}_t,\ t \in \mathcal{H},\ \omega \in \Omega_t,$$

$$\sum_{k \in \mathcal{K}^{\mathsf{RES}}} V_{n,k} \geq \varphi_t \sum_{k \in \mathcal{K}} V_{n,k}, \quad n \in \mathcal{N}_t,\ t \in \mathcal{H}.$$

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Case study: the Italian Power System

- **4 zones**: North, South, Sicily, and Sardinia, connected by 4 transmission lines;
- **4 generation technologies**: CCGT, GT, photovoltaic and wind power;
- **3 storage technologies**: pumped-hydro storage, lithium-ion batteries and sodium-ion batteries;
- **3 target years**: 2025, 2030, 2040.

	Small Instance	Large Instance
Strategic Stages	2	5
Strategic Nodes	3	31
Strategic Scenarios	2	16
Operational Scenarios per Strategic Node	64	64
Number of constraints	426,299	9,637,779
Number of variables	350,449	7,284,977
CPU time (s)	4,635	-

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Numerical Results: Lower bounds via expectations and upper bounds

	Lower Bounds	Small Instance	Large Instance
$MHEV$	Percentage Gap	-27.34%	-11.99%
	CPU Time (s)	2	5
$MHOEV$	Percentage Gap	-24.49%	-10.93%
	CPU Time (s)	3	9
$MHSEV$	Percentage Gap	-17.66%	-7.81%
	CPU time (s)	204	1121

Large Instance	Upper Bound (bln €)	CPU time (s)
Fixing Approach		
$MHESS_2$	307.05	3211
$MHESS_1$	298.59	4541
Rolling Horizon Approach	286.93	15153

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Numerical Results: Small Instance (disjoint groups)

		$MHEG(\ell, \gamma)$	Percentage Gap: Level of strategic chain ℓ = 2 (1 group)	Percentage Gap: Level of strategic chain ℓ = 1 (2 groups)	CPU Time (s): Level of strategic chain ℓ = 2 (1 group)	CPU Time (s): Level of strategic chain ℓ = 1 (2 groups)
Level of **operational** chain γ	7	(1 group)	0.00%	-2.01%	4635	407
	6	(8 groups)	-0.06%	-2.07%	1944	1512
	5	(64 groups)	-0.15%	-2.15%	1784	1124
	4	(512 groups)	-0.40%	-2.42%	2705	1905
	3	(4096 groups)	-1.21%	-3.15%	6581	6488
	2	(32768 groups)	–	–	–	–
	1	(262144 groups)	–	–	–	–

		$MHNG(\ell, \gamma)$	Percentage Gap: Level of strategic chain ℓ = 2 (1 group)	Percentage Gap: Level of strategic chain ℓ = 1 (2 groups)	CPU Time (s): Level of strategic chain ℓ = 2 (1 group)	CPU Time (s): Level of strategic chain ℓ = 1 (2 groups)
Level of **operational** chain γ	7	(1 group)	0.00%	-2.01%	4635	407
	6	(2 groups)	-0.09%	-2.09%	518	403
	5	(4 groups)	-0.16%	-2.18%	346	218
	4	(8 groups)	-0.63%	-2.58%	201	112
	3	(16 groups)	-1.85%	-3.88%	71	70
	2	(32 groups)	-3.83%	-5.82%	62	64
	1	(64 groups)	-6.10%	-8.14%	52	50

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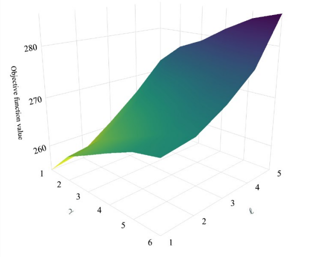
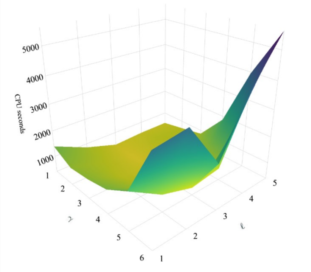
Numerical Results: Large Instance (disjoint groups)

Best upper bound: 286.93

Lower Bounds



CPU Time



		Percentage Gap					CPU Time (s)				
		Level of **strategic** chain ℓ					Level of **strategic** chain ℓ				
	$MHNG(\ell, \gamma)$	5	4	3	2	1	5	4	3	2	1
		(1 group)	(2 groups)	(4 groups)	(8 groups)	(16 groups)	(1 group)	(2 groups)	(4 groups)	(8 groups)	(16 groups)
Level of **operational** chain γ	7 (1 group)	–	-3.24%	-5.03%	-6.44%	-6.71%	–	7629	6073	6113	6495
	6 (2 groups)	**-0.34%**	-3.63%	-5.39%	-6.71%	-7.10%	5587	4128	2065	3742	3568
	5 (4 groups)	-0.84%	-4.10%	-5.79%	-7.23%	-7.53%	3953	826	651	1024	1782
	4 (8 groups)	-1.80%	-4.94%	-6.64%	-8.14%	-8.46%	1858	706	463	762	1278
	3 (16 groups)	-2.96%	-5.73%	-7.49%	-9.09%	-9.47%	853	544	423	629	1108
	2 (32 groups)	-3.78%	-6.32%	-8.24%	-10.01%	-10.48%	685	704	373	719	1023
	1 (64 groups)	-5.35%	-7.79%	-9.94%	-11.95%	-12.60%	528	524	462	847	1395

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Case study 2: Domestic Renewable Energy System Design

- The problem consists in planning investments in **PV panels** and **batteries** to supply electricity to a residential building complex.
- **Decision variables**:
  1. **strategic**: The capacity of PV panels and batteries in the long-term;
  2. **operational**: Scheduling of generation, storage, and electricity exchanegs with the grid to supply an elastic demand for electricity in every hour of representative days.
- **Uncertain parameters**:
  1. **Strategic**: investment and maintenance costs;
  2. **Operational**: solar radiation, electricity prices, and electricity demand.

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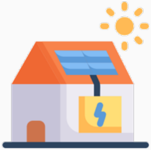
Experimental Setup: real data from a German building complex

- **Location:** Southern Germany
- **PV Options:** 3 types
- **BESS Options:** 2 types
- **Data Sources:** ENTSOE, CoSSMic



	Small	Medium	Large
Strategic stages	3	4	6
Strategic nodes	13	40	364
Strategic scenarios	9	27	243
Operational scenarios per node	10	20	20
Number of constraints	158,565	1,402,311	21,606,345
Number of continuous variables	46,970	384,921	6,116,293
Number of binary variables	47,060	433,200	5,252,540
Objective function value (€)	5,746,672	13,627,470	–
CPU time (s)	35	1,016	–

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Numerical Results

Lower Bounds (Large Instance)

Model	Lower Bound (€)	Time (s)
$MHEV$	20,218,777	46
$MHOEV$	24,981,335	192
$MHC(5)$	60,499,195	983
$MHC(4)$	60,562,808	721
$MHC(3)$	60,563,201	705
$MHC(2)$	**60,565,811**	841

Upper Bounds (all instances)

Instance	$\hat{e}$	$\hat{e}^R$	ϕ_e	GAP	Time (s)
Small	1	0	0	2.27%	29
Small	2	0	0	0.38%	32
Small	1	2	$\frac{1}{3}$	**0.06%**	31
Small	2	1	$\frac{1}{3}$	**0.06%**	34
Medium	1	0	0	3.57%	136
Medium	2	0	0	2.05%	218
Medium	1	2	$\frac{1}{3}$	0.83%	306
Medium	2	1	$\frac{1}{3}$	**0.66%**	261
Large	1	0	0	5.70%	570
Large	2	0	0	3.48%	657
Large	3	0	0	2.61%	2,593
Large	2	2	$\frac{1}{3}$	0.70%	789
Large	3	1	$\frac{1}{3}$	**0.69%**	3,098

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Conclusions and future works

In this talk, **bounds multi-horizon SO** problems have been described:

- **Strategic** and **operational expectation based** bounds.
- **Strategic** and **operational scenario tree decomposition** bounds.

The bounds do not require any structural properties and are applicable to a **broad class of problems**.

Current research directions:

- Integrate bounds with **decomposition methods** (Benders' decomposition).
- Provide bounds for **distributionally robust multi-horizon optimization**.
- Provide bounds for **decision-dependent stochastic optimization**.
- Integrate **Neural networks** to solve MHSO problems.

References

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