

The curse of high dimensionality

Our models have high dimensionality:



Many considered years $[\mathcal{O}(10)]$ and time steps $[\mathcal{O}(1000)]$



Many nodes $[\mathcal{O}(100)]$



Many energy carriers $[\mathcal{O}(10)]$ and technologies $[\mathcal{O}(100)]$



Complex technology descriptions (binary operations, nonlinear relationships)



Scenarios and uncertainty handling

We need to reduce complexity without sacrificing (much) accuracy:

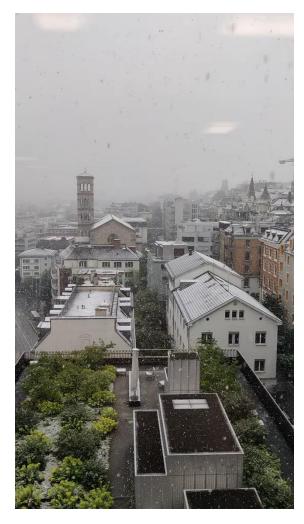
The most established technique is time series aggregation (TSA)

Utilizing the fact that many hours in the year show similar conditions

The complexities we are discussing in this workshop!



Why do we want to do time series aggregation?



April 18th, 2024, at 10 am



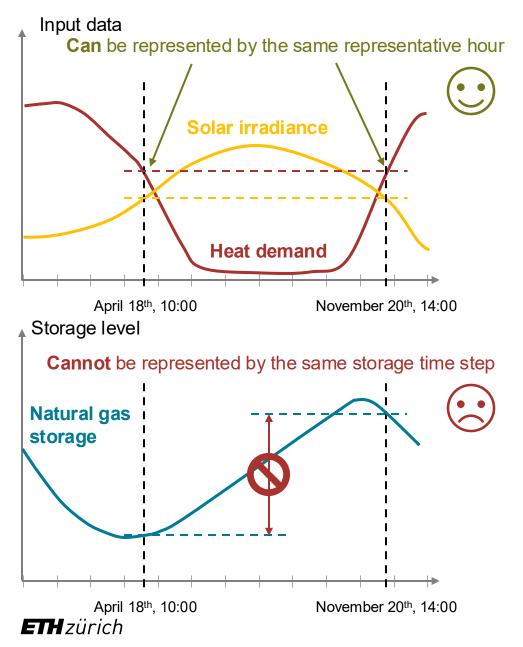
November 20th, 2025, at 2 pm

We need to reduce complexity without sacrificing (much) accuracy:

The most established technique is time series aggregation (TSA)

Utilizing the fact that many hours in the year show similar conditions

What is the issue with time series aggregation?

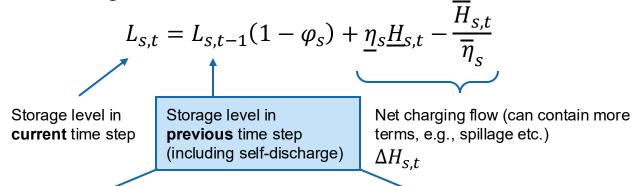


Storage representation problem:

The storage level is time-coupled, i.e., the storage level depends on the previous time step

Let's formalize the problem:

The storage level is formulated as:



TSA loses chronology of time steps

→ "previous" time step does not exist anymore

Storage representation must rebuild the storage level chronology

What do we want to do today?



Compare clustering representative days (RD) and representative hours (RH)



Highlight the **limitations** of current RH storage representation methods



Present the novel RH storage representation method for large-scale multi-storage energy systems



Benchmark existing methods in terms of accuracy vs computational complexity



Representative days (RD) vs representative hours (RH)

The most common way to solve the storage representation problem [1]:

- Keep daily chronology of the clustered time steps
 representative days (RD)
- For long(er)-term storage:
 Superpose daily storage levels with the storage levels of the RD

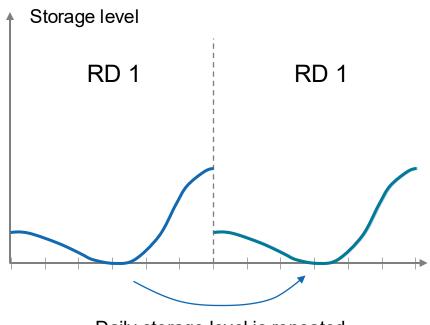
However, ...

Clustering RD requires many representative time steps to approximate the input data well!



Representative hours (RH) require fewer time steps for the same approximation quality





Daily storage level is repeated

Why are RH storage representation methods underrepresented?

Until now, the storage level cannot be reconstructed accurately and efficiently

Limitations of current RH storage representation methods

Reconstructing the storage level chronology is especially challenging for non-chronological RH

Fully resolved storage level [1]

- → Idea: Resolve the storage level with 8760 time steps per year → Fully resolved storage level
- ✓ Chronology: yes!
- <u>Limitation</u>: Introduces many additional variables and constraints
 → computationally challenging

State-space transitions [2]

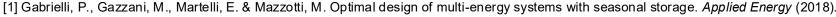
- transition from one RH to another with central differences
- Chronology: yes!
- Limitation: Central differences approximate and smoothen storage level

Chronological RH [3]

- ☐ Idea: Cluster chronological RH
 → storage level time steps are the operational time steps
- Chronology: yes!
- Limitation: Strongly smoothens daily variability

Proposed reduced RH storage representation

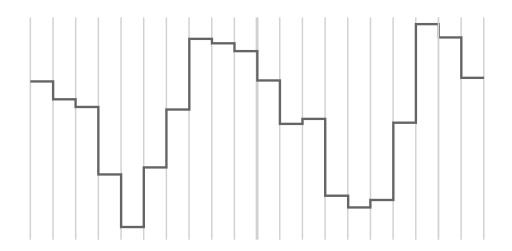
<u>Idea</u>: Resolve the storage level but only keep the necessary storage level time steps



^[2] Wogrin, S., Galbally, D. & Reneses, J. Optimizing Storage Operations in Medium- and Long-Term Power System Models. *IEEE Transactions on Power Systems* (2016).

^[3] Pineda, S. & Morales, J. M. Chronological Time-Period Clustering for Optimal Capacity Expansion Planning With Storage. IEEE Transactions on Power Systems (2018).

Let's take a generic input time series with 20 time steps

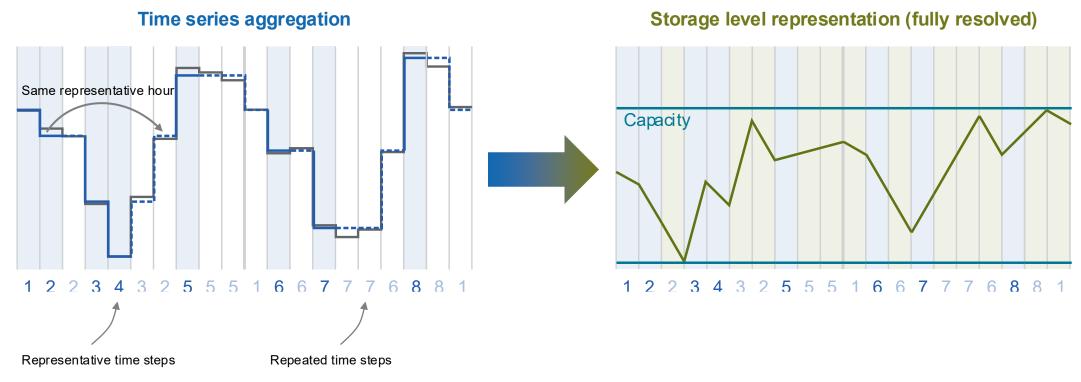




We cluster the input time series into 8 representative time steps

Time series aggregation Same representative hour 1 2 2 3 4 3 2 5 5 5 1 6 6 7 7 7 6 8 8 1 Representative time steps Repeated time steps

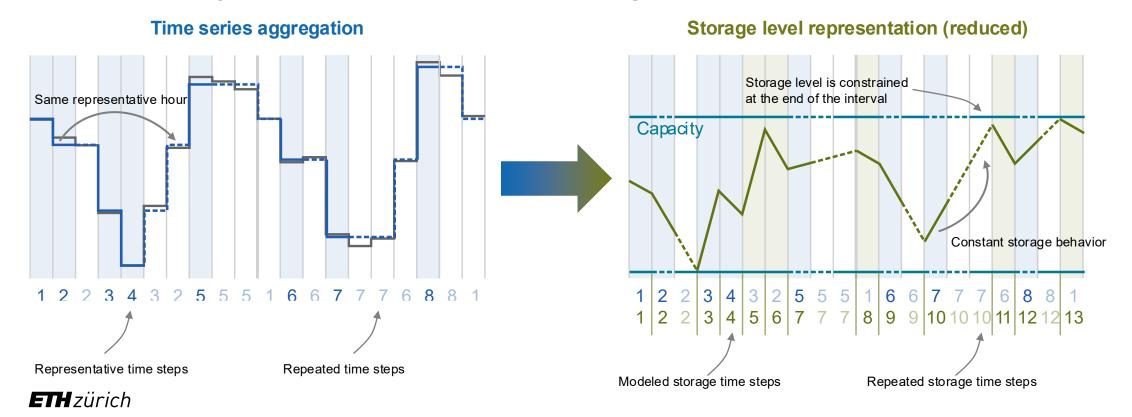
We can reconstruct the storage level at every time step [1] But can we reduce the number of storage level time steps? **Yes!**



Observation: The storage behavior does not change over the same representative hour

- → For every cluster of adjacent hours represented by the same RH: one storage time step
- → Every time we change the representative time step, we add one storage level time step
- → We can save many variables and constraints (from 20 to 13 storage level time steps)

We don't lose any information compared to full storage resolution!

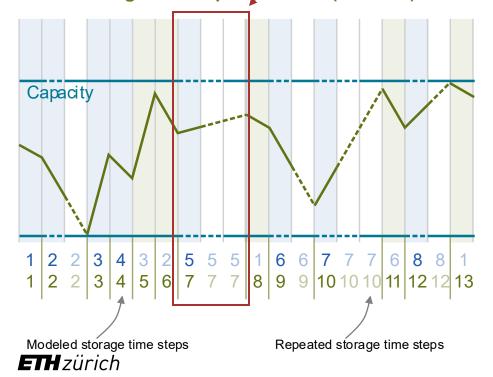


Mathematical description of reduced RH storage representation with self-discharge

New indices:

Representative time steps: $i \in I$ Storage level time steps: $i \in I$

Storage level representation (reduced)



Constant net charging for RH i = 5:

$$\Delta H_{s,t=8} = \Delta H_{s,t=9} = \Delta H_{s,t=10} = \Delta H_{s,i=5}$$

Storage level equations for t = 8, t = 9, and t = 10

$$L_{s,t=8} = L_{s,t=7}(1 - \varphi_s) + \Delta H_{s,i=5}$$

$$L_{s,t=9} = L_{s,t=8}(1 - \varphi_s) + \Delta H_{s,i=5}$$

$$L_{s,t=10} = L_{s,t=9}(1 - \varphi_s) + \Delta H_{s,i=5}$$

We always add the same $\Delta H_{s,i=5}$!

$$L_{s,t=10} = L_{s,t=7} (1 - \varphi_s)^3 + \Delta H_{s,i=5} \sum_{\tilde{j}=0}^{2} (1 - \varphi_s)^{\tilde{j}}$$

$$L_{s,j=7} \qquad L_{s,j=6}$$

Generic storage level constraint:

$$L_{s,j} = L_{s,j-1} (1 - \varphi_s)^{d_j} + \Delta H_{s,\vartheta(j)} \sum_{\tilde{j}=0}^{d_j-1} (1 - \varphi_s)^{\tilde{j}}$$

Duration of storage level time step

Unique mapping

Benchmarking on a greenfield optimization model of the European electricity and heating system

We benchmark the proposed method against four established methods:

- 1. Superposition (RD) [1]
- 2. MinMax (RD) [1]
- 3. Full storage resolution (RH) [2]
- 4. Chrono (Chronological RH) [3]

Only compare against methods that do not approximate the storage level

Optimize a **greenfield cost-minimization model** of the European (28 countries) electricity and heating system

All storage representation methods are implemented in ZEN-garden [4] with hierarchical clustering (except for Chrono) and mean representation

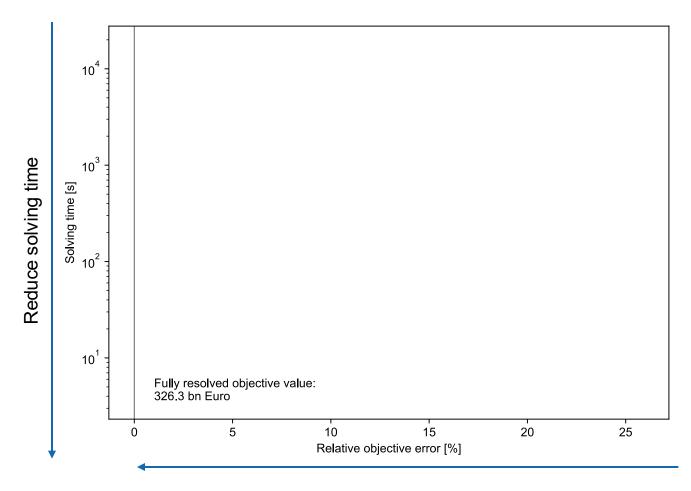
garden

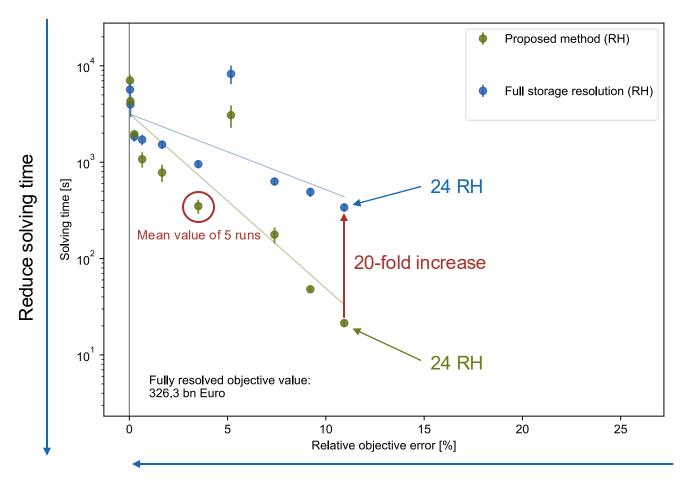
Anniliad France (0010)

^[1] Kotzur, L., Markewitz, P., Robinius, M. & Stolten, D. Time series aggregation for energy system design: Modeling seasonal storage. Applied Energy (2018).

^[2] Gabrielli, P., Gazzani, M., Martelli, E. & Mazzotti, M. Optimal design of multi-energy systems with seasonal storage. Applied Energy (2018).

^[3] Pineda, S. & Morales, J. M. Chronological Time-Period Clustering for Optimal Capacity Expansion Planning With Storage. *IEEE Transactions on Power Systems* (2018). ETH ZUTICh [4] Mannhardt, J. et al. ZEN-garden: Optimizing energy transition pathways with user-oriented data handling. *SoftwareX* (2025).





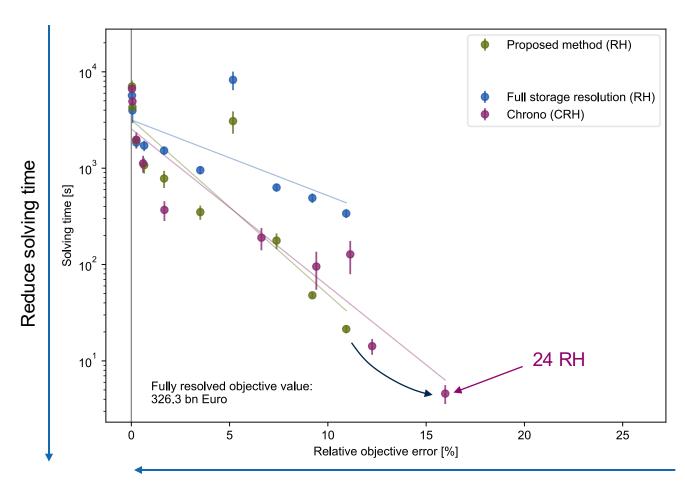
The proposed method shows lower solving times than *Full storage resolution* at the same objective error

Up to 20-fold solving time increase for *Full storage resolution*.

The strongest aggregation (24 RH) leads to an objective error of ~10%





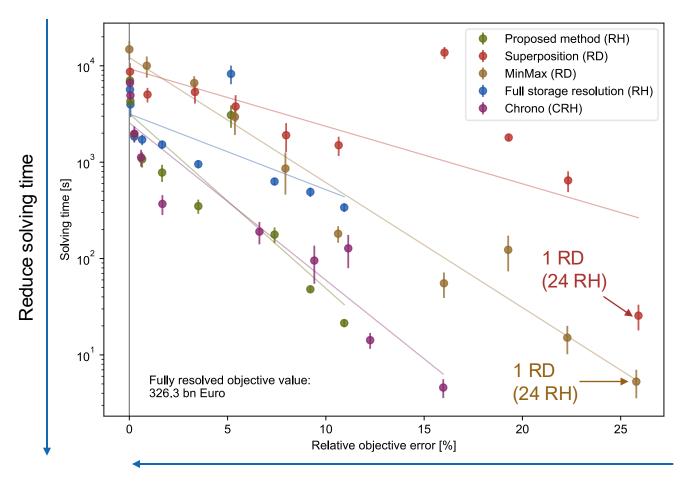


Chrono shows similar accuracy and solving time as the proposed method

At the strongest aggregation, increased objective error (16%)







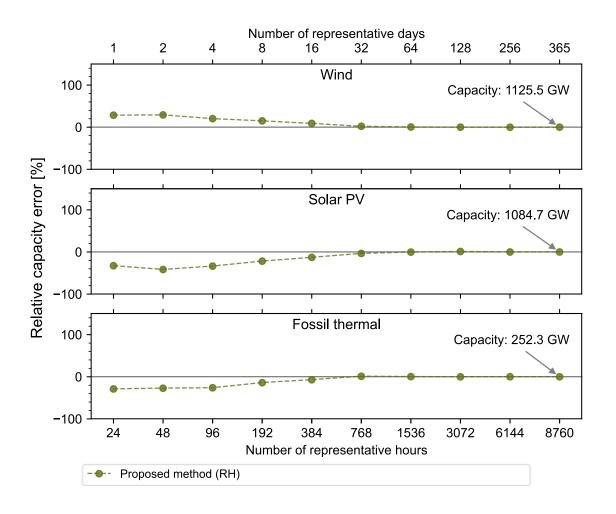
The RD methods show the highest solving time and the highest objective error.

MinMax can reduce the solving time compared to Superposition

Reduce objective error



RH methods show a smooth reduction of the capacity error; Negligible error after 768 RH



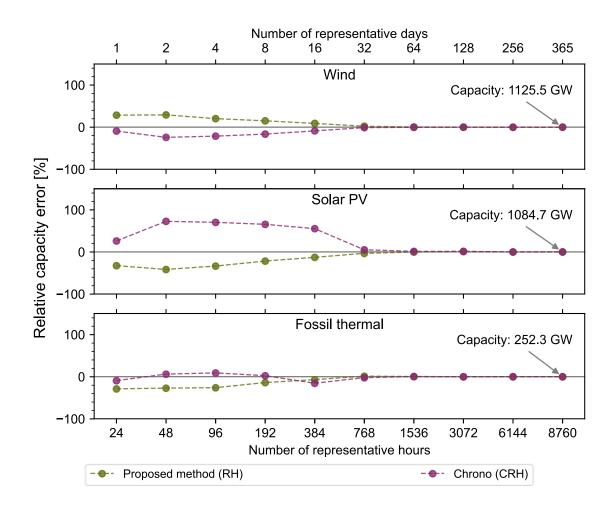
The proposed method overestimates on shore wind and underestimates solar PV

Maximum capacity error: solar PV: - 40% (48 RH)

Lower fossil thermal capacity because time series are smoothed



RH methods show a smooth reduction of the capacity error; Negligible error after 768 RH



Chrono shows significantly higher solar PV capacity

Reason: daily profiles are removed entirely for strong aggregation

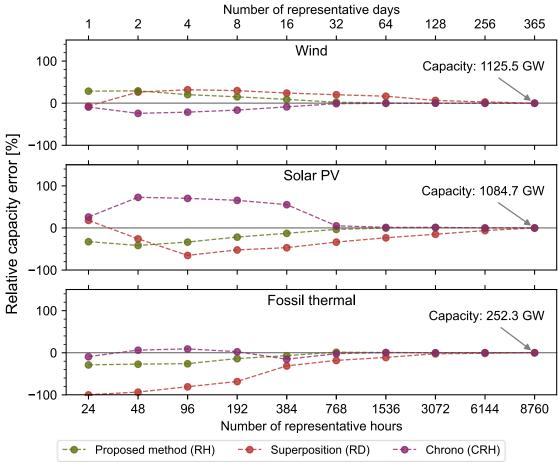
→ solar PV is more viable

At 768 RH: roughly 2 RH per day

- → daily behavior is captured
- → good approximation of solar PV



RH methods show a smooth reduction of the capacity error; Negligible error after 768 RH



Superposition shows a persistent capacity error

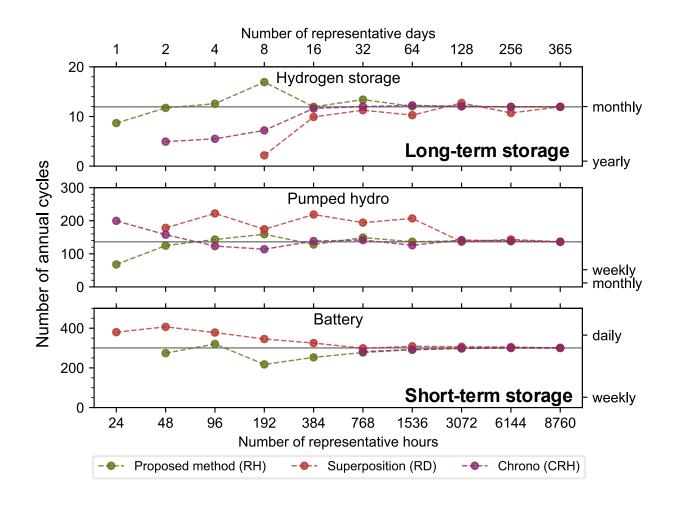
Especially, in the required fossil thermal capacity

For solar PV, inverse effect of *Chrono*: Only daily profile

→ lower viability of solar PV



The proposed method approximates the storage behavior well



The proposed method shows a good approximation of the storage behavior, even at strong aggregation

Chrono shows lower hydrogen storage cycles and negligible battery capacity until 768 RH

Superposition underestimates the use of hydrogen storage and overestimates the use of batteries



Conclusions



Contribution: RH-based storage level representation method with reduced storage level index

Clustering accuracy of using representative hours



Storage level representation to ensure computational tractability

Conclusions:



RH represent the input data time series more accurately than RD with fewer representative time steps



The proposed method shows a **strong reduction in solving time** while retaining a good accuracy



The method can reproduce the operation of short-term and long-term storage well



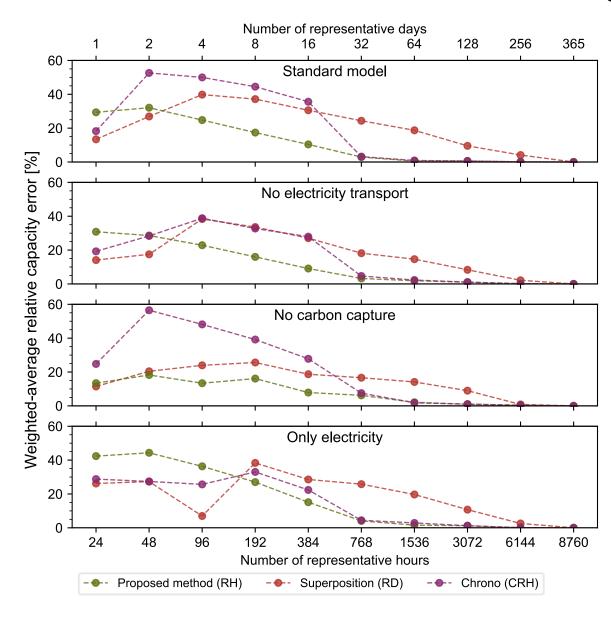
100 to 500 representative hours are a good trade-off between accuracy and computational complexity for large-scale multi-storage energy systems



Appendix

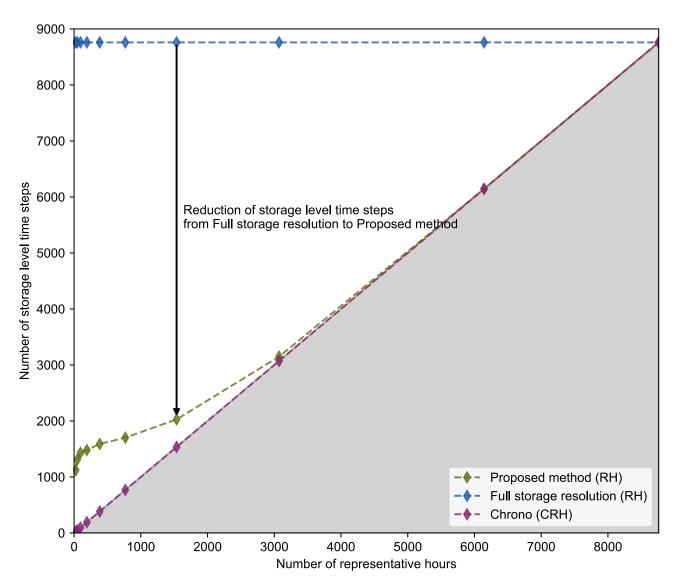


Robustness of methods for various large-scale energy system models



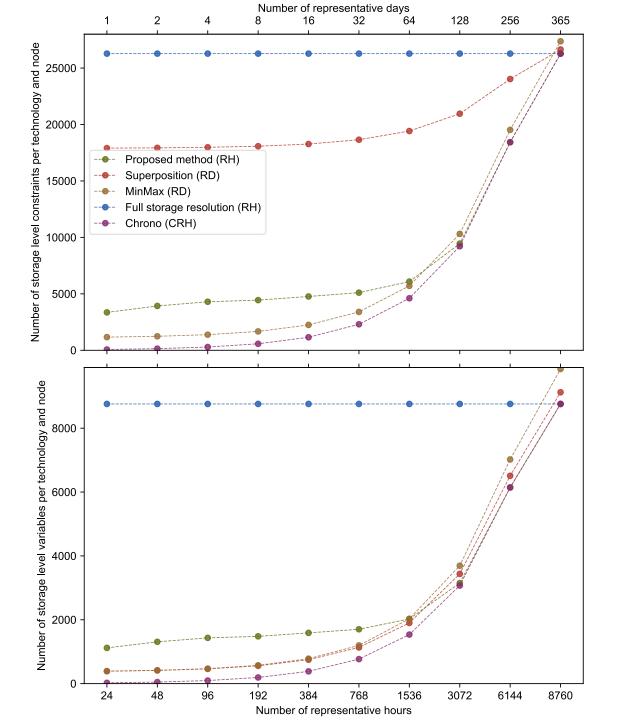


Number of time steps for RH methods



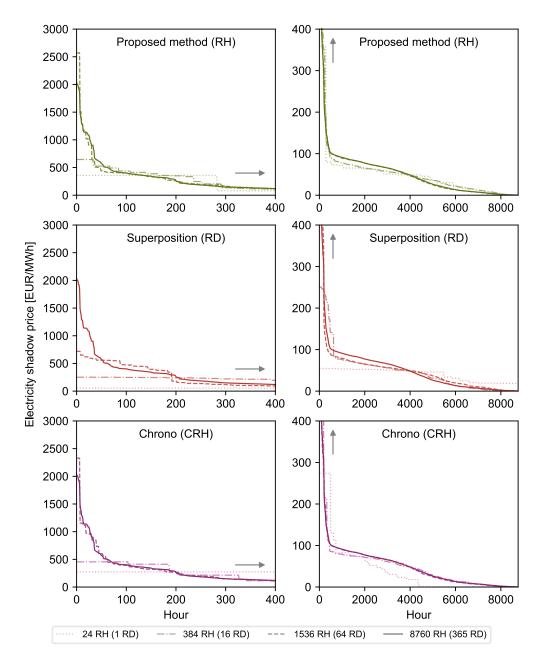


Number of constraints and variables

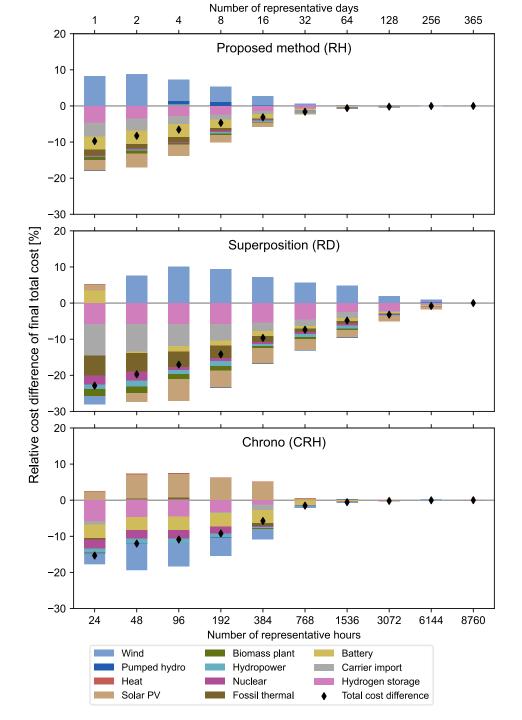




Price duration curve of the shadow price of the electricity energy balance

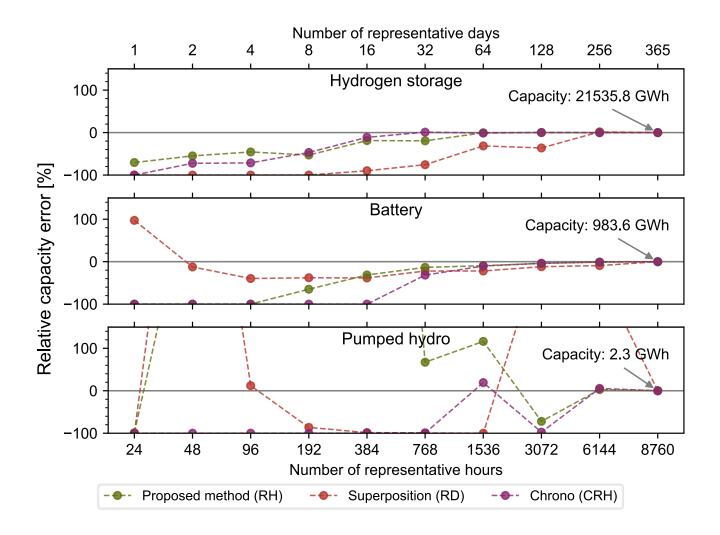


Cost error



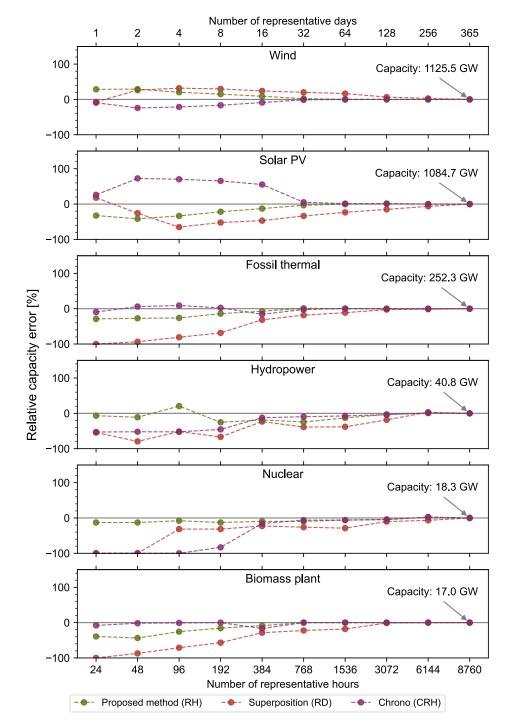


Capacity error storage



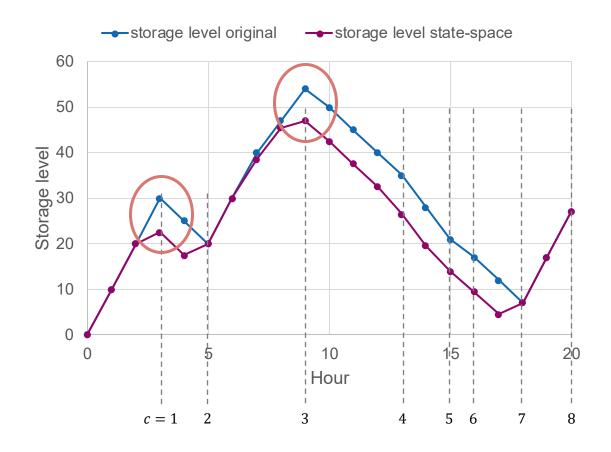


Capacity error conversion





State-space transition reconstruction



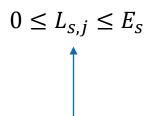
$$\Delta L_{s,k} = 0.5 \left(\Delta H_{s,\kappa(k)} + \Delta H_{s,\kappa'(k)} \right)$$
, where $k = (i,i')$
 $\kappa: \mathcal{K} \to \mathcal{I}, \kappa': \mathcal{K} \to \mathcal{I}'$ (Transitions)

$$L_s^0 + \sum_{k \in \mathcal{K}} \Delta L_{s,k} \, F_{k,c} \ge 0$$

$$L_s^0 + \sum_{k \in \mathcal{K}} \Delta L_{s,k} F_{k,c} \ge 0$$

$$L_s^0 + \sum_{k \in \mathcal{K}} \Delta L_{s,k} F_{k,c} \le E_s$$

Storage level monotony



Is $L_{s,j}$ monotonic over j? Yes!

$$L_{\hat{t}} = L_0 (1 - \varphi)^{\hat{t}} + \Delta H \sum_{\tilde{t}=0}^{\hat{t}-1} (1 - \varphi)^{\tilde{t}}$$
 for the intermediate time steps $\hat{t} \in [1, d_j]$

Two cases:

$$\varphi = 0$$

$$\frac{\mathrm{d}L_{\hat{t}}}{\mathrm{d}\hat{t}} = \Delta H$$

$$\frac{\mathrm{d}L_{\hat{t}}}{\mathrm{d}\hat{t}} = \underbrace{\left(L_0 - \frac{\Delta H}{\varphi}\right) \ln(1 - \varphi)}_{= \text{ constant } \forall \hat{t} \in [1, d_j]} (1 - \varphi)^{\hat{t}}$$

Can't change its sign over $\hat{t} \in [1, d_j] \rightarrow$ monotonic